ProbGraph: High-Performance and High-Accuracy Graph Mining with Probabilistic Set Representations

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Motivation

- Graph mining is slow
    - Hard to parallelize since there exists little locality and irregularities in some graphs
- Useful to many problems in modern graphs
    - Examples: Triangle Counting, Clique Counting, Vertex Similarity, Graph Clustering

Contributions

- Provides an approximate algorithm trading accuracy for speed
- Helps general class of graph problems requiring set intersections in their routines.
- Approximation is tunable, and claims up to 50x speedups with up to 90% accuracy

Data Review

- Claims appear to lack support in their data, there is high variance, and not a clear link on how they get 98% or 90% accuracy claims.

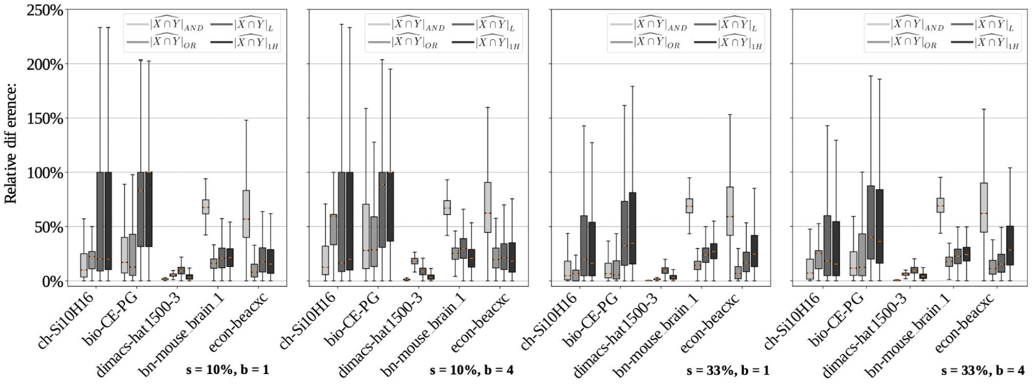
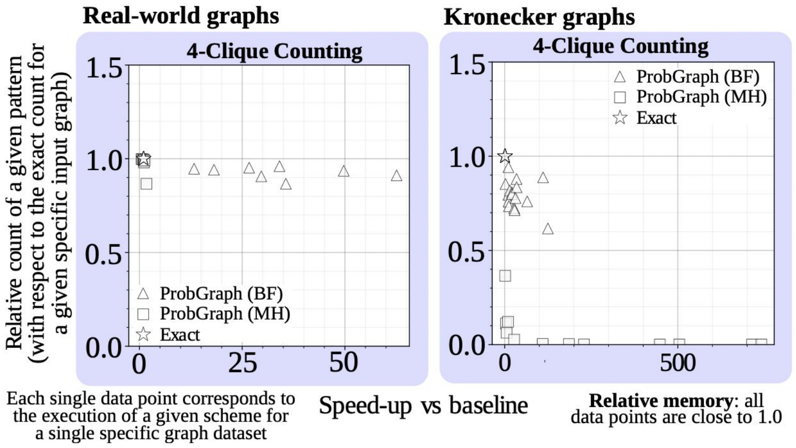
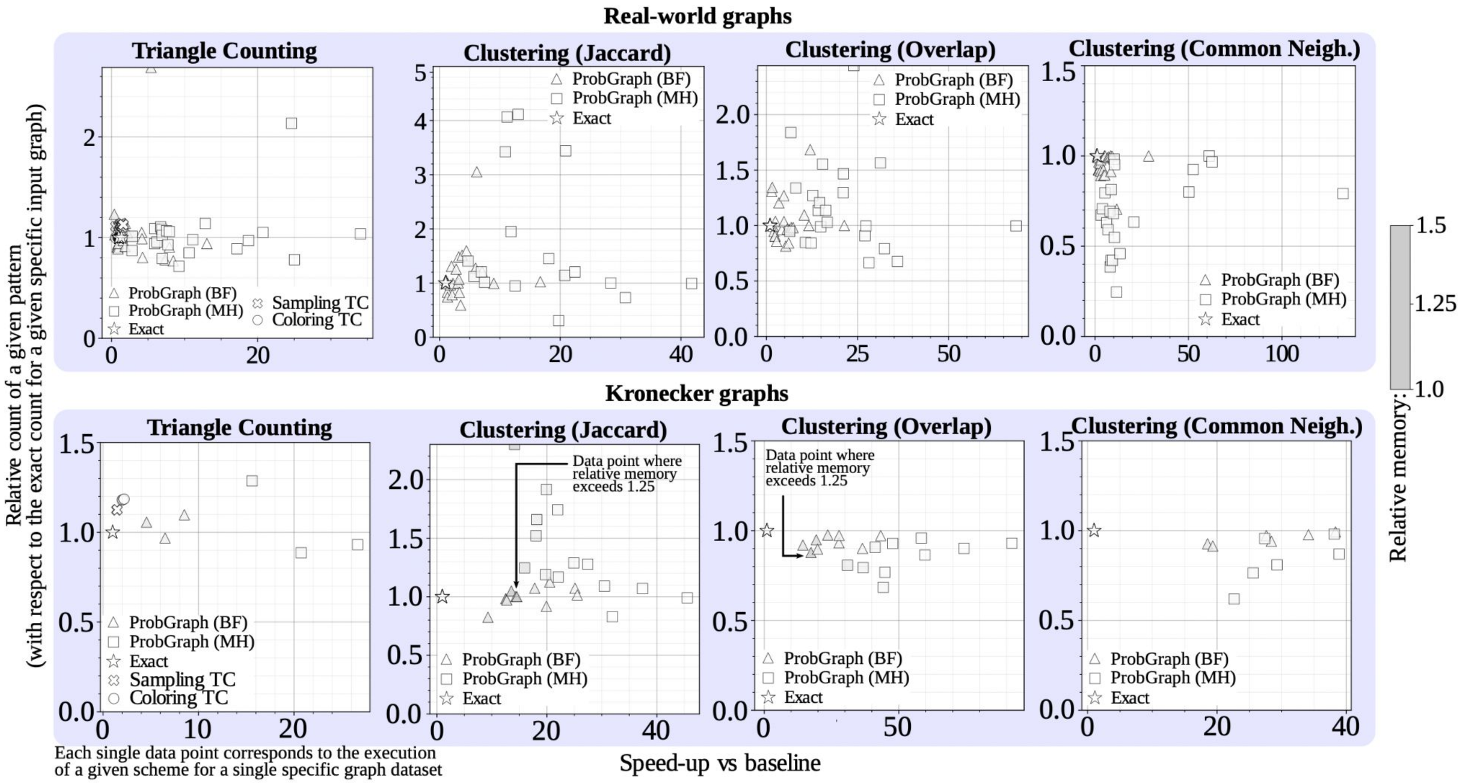


Fig. 3: Analysis of the accuracy of PG estimators of $|X \cap Y|$.



Additional Data Review

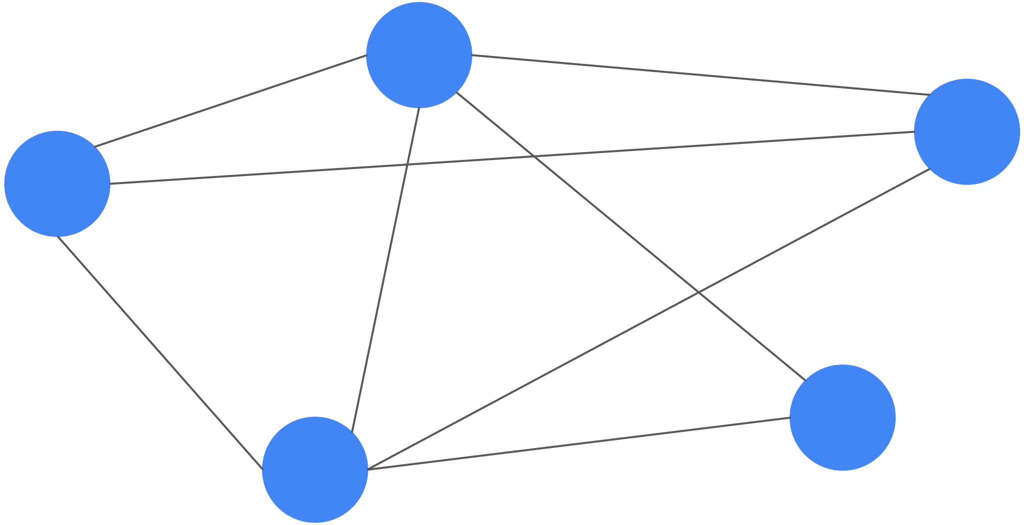


Overview

- Provide background on triangle counting to use as a motivating example
- Recognize a common subroutine in computation is set intersections
- Delve into Bloom Filters and MinHash approximation algorithms
- Show approximation algorithm given in ProbGraph

Triangle Counting

- Find all unique triples such that each pair of vertices shares and edge.
- Used to analyze real world graphs: cluster coefficient, spam filtering, find structure
- There is an n^3 algorithm: enumerate all triples and check



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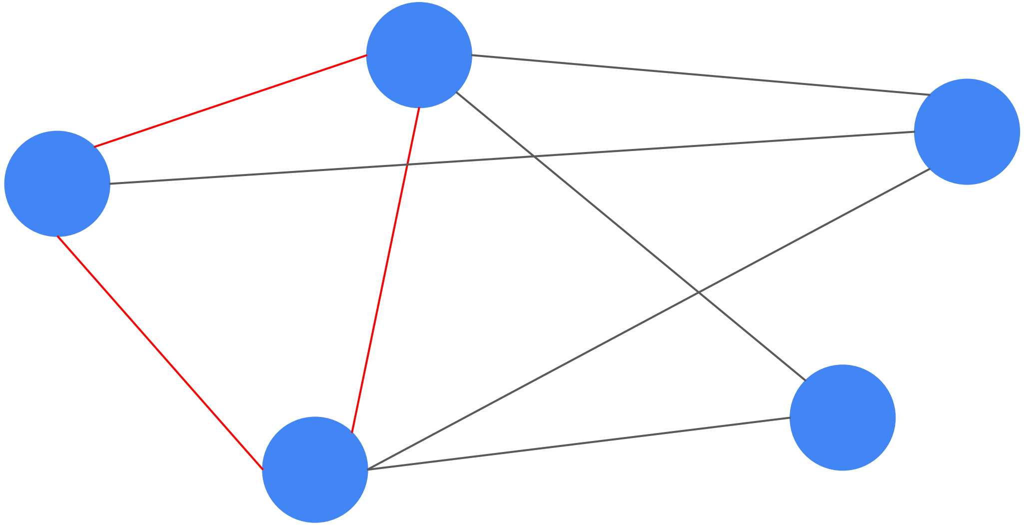

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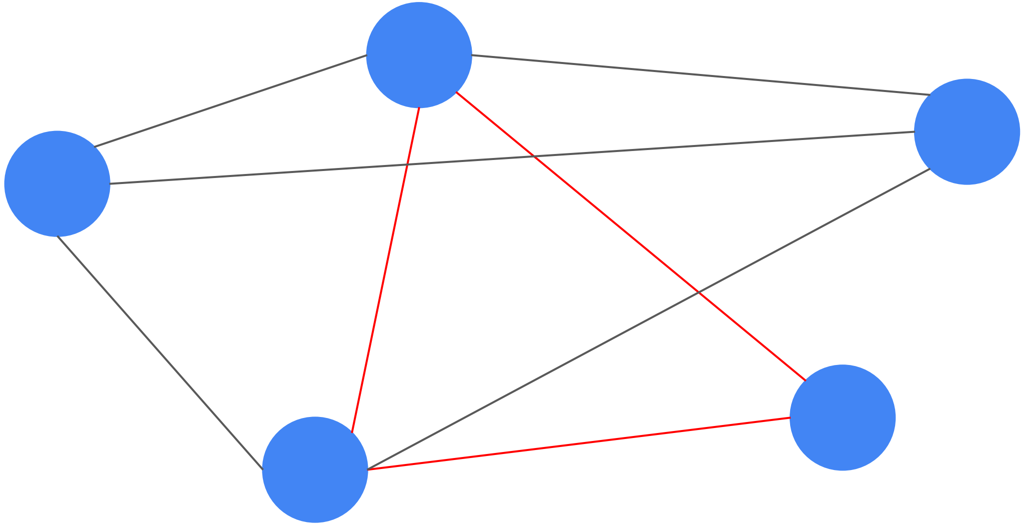

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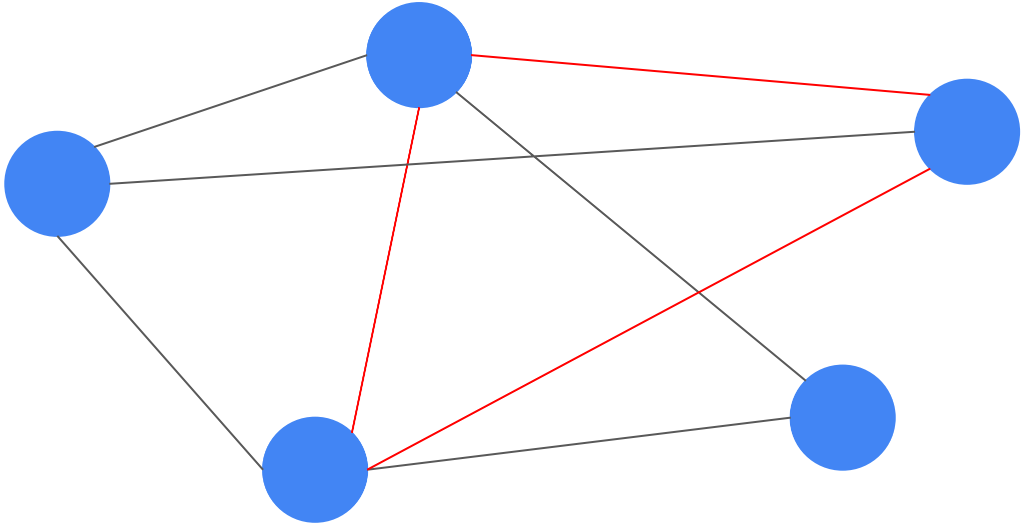

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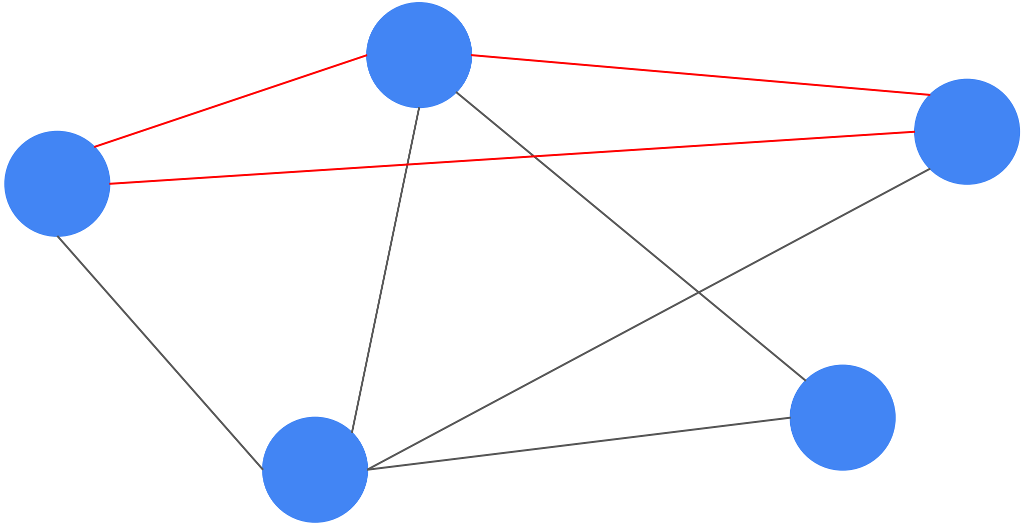

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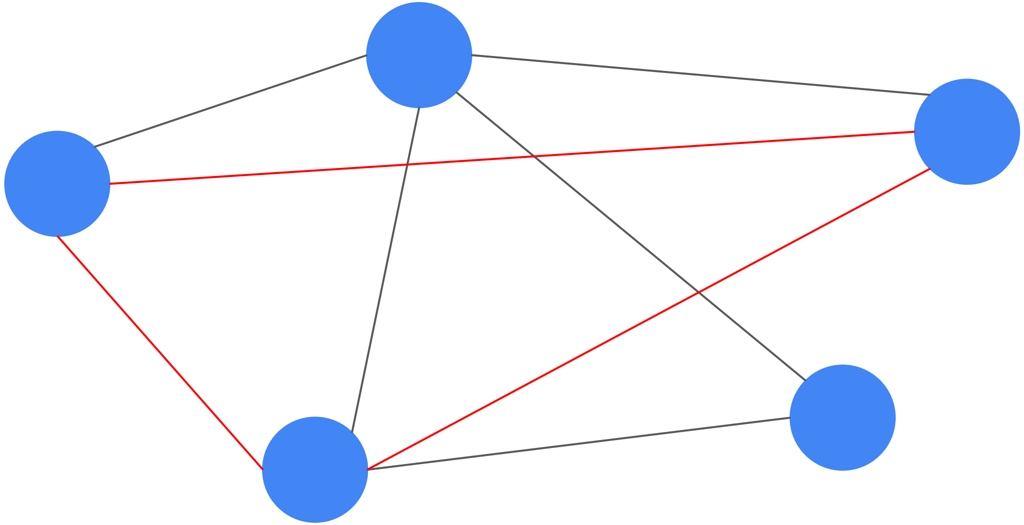
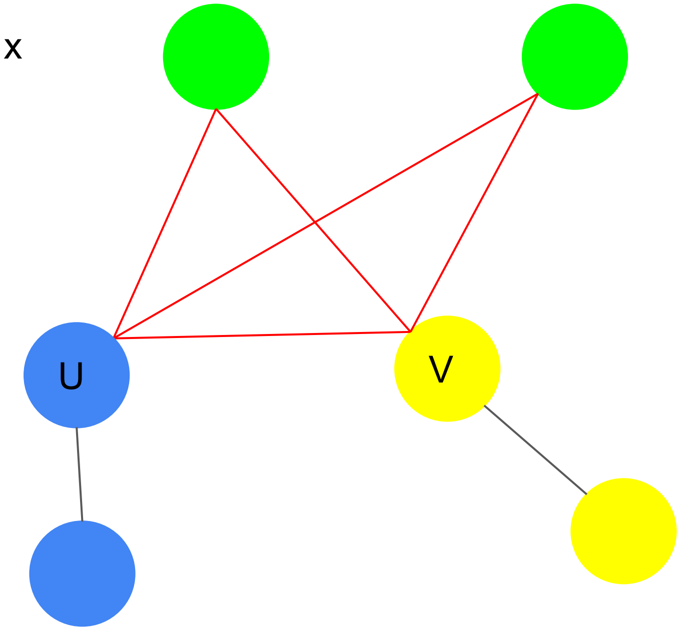


Triangle Counting Faster Approach

- Let U, V be neighboring vertices and N_x be the neighbors of x
- Then $N_U \cap N_V$ / {U, V} are triangles

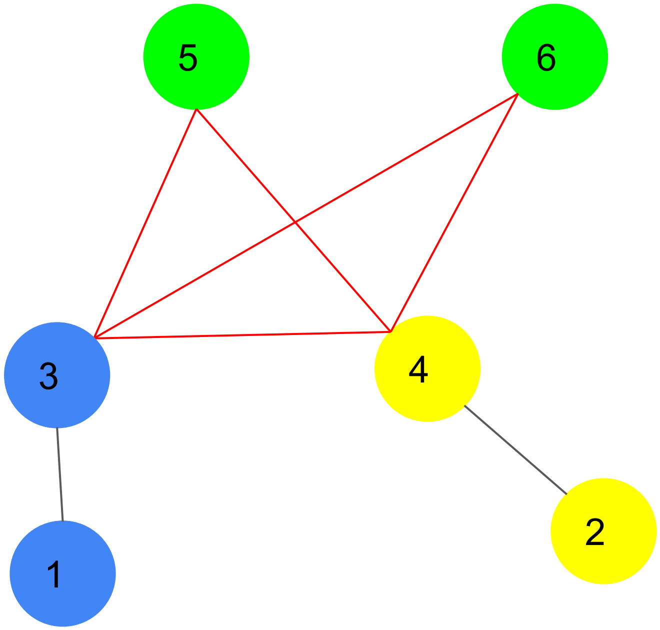


Triangle Counting Algorithm

// Derive a vertex order R s.t if $R(v) < R(u)$ then $d_v \le d_u$
for $v \in V$ do: $N_v^+ = \{u \in N_v \mid R(v) < R(u)\}$

tc = 0
for $v \in V$ do:
    for $u \in N_v^+$ do: tc += $|N_v^+ \cap N_v^+|$

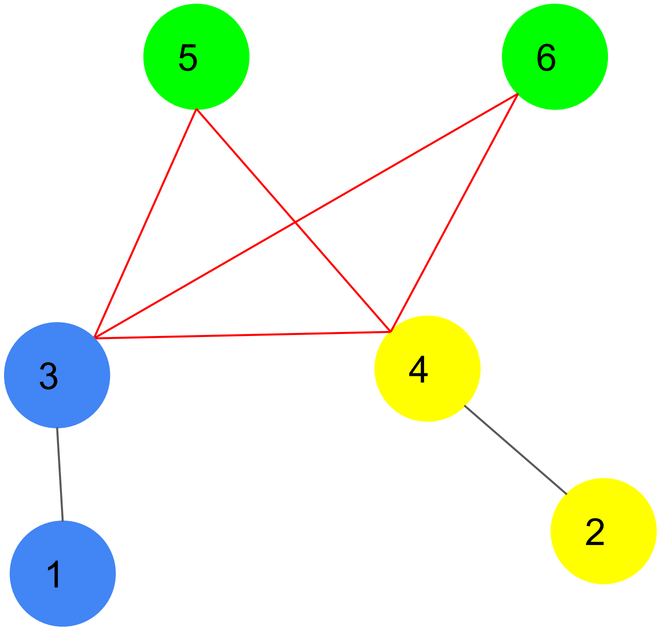
- Let d be max degree and n be number of nodes
- Initial loop takes O(nd)
- Main loop takes $O(nd^2)$



Triangle Counting Parallel Algorithm

// Derive a vertex order R s.t if $R(v) < R(u)$ then $d_v \leq d_u$
for $v \in V$ [in par] do: $N_v^+ = \{u \in N_v \mid R(v) < R(u)\}$

tc = 0
for $v \in V$ [in par] do:
    for $u \in N_v^+$ [in par] do: tc += $|N_v^+ \cap N_v^+|$



Other examples

- Clique Counting
- Vertex Similarity
- Graph Clustering

```
/* Input: A graph G. Output: Number of 4-cliques ck ∈ ℕ. */
/Derive a vertex order R s.t. if R(v) < R(u) then d_v ≤ d_u:
for v ∈ V [in par] do: N_v^+ = {u ∈ N_v | R(v) < R(u)}
ck = 0;
for u ∈ V [in par] do:
  for v ∈ N_u^+ [in par] do:
    C_3 = N_u^+ ∩ N_v^+ //Find 3-cliques
    for w ∈ C_3 do: //For each 3-clique...
      ck += |N_w^+ ∩ C_3| //Find 4-cliques
```

Listing 2: Reformulated 4-Clique Counting.


```
/* Input: A graph G. Output: Number of 4-cliques ck ∈ ℕ. */
/Derive a vertex order R s.t. if R(v) < R(u) then d_v ≤ d_u:
for v ∈ V [in par] do: N_v^+ = {u ∈ N_v | R(v) < R(u)}
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```

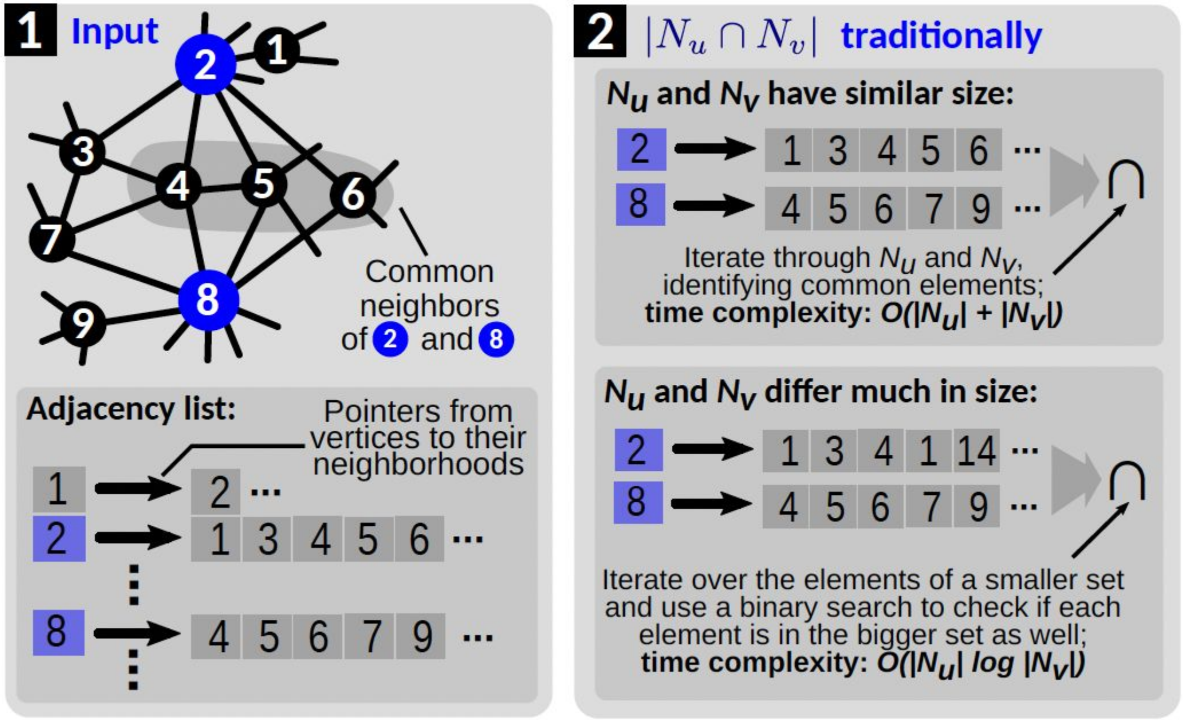
Listing 2: Reformulated 4-Clique Counting.


```
/* Input: A graph G = (V, E). Output: Clustering C ⊆ E
 * of a given prediction scheme. */
//Use a similarity S_C(v, u) = |N_v ∩ N_u| (see Listing 3).
for e = (v, u) ∈ E [in par] do: //τ is a user-defined threshold
  if |N_v ∩ N_u| > τ: C ∪= {e}
//Other clustering schemes use other similarity measures.
```

Listing 4: Jarvis-Patrick clustering.

Bottleneck

- |X ∩ Y| is slow



How to make $|X \cap Y|$ faster?

- Trading some accuracy for speed
- Use of Bloom Filters and MinHash sets to approximate these intersections

Bloom Filter

- Want space efficient/fast answering to membership queries
- False positives
- Bloom filter has L element bit vector
  - Set of hashes, $\{h_i\}$, computes an integer in [1, L]
- Add Element
  - Compute each hash, set corresponding bit to 1
- Retrieve Element
  - Compute each hash, if all 1, return True

2 hashes, $L \in [1, 3]$

Insert a -> {1, 1}
BF = 100
Insert b -> {3}
BF = 101

Now c -> {1, 3} would be “contained” although not inserted

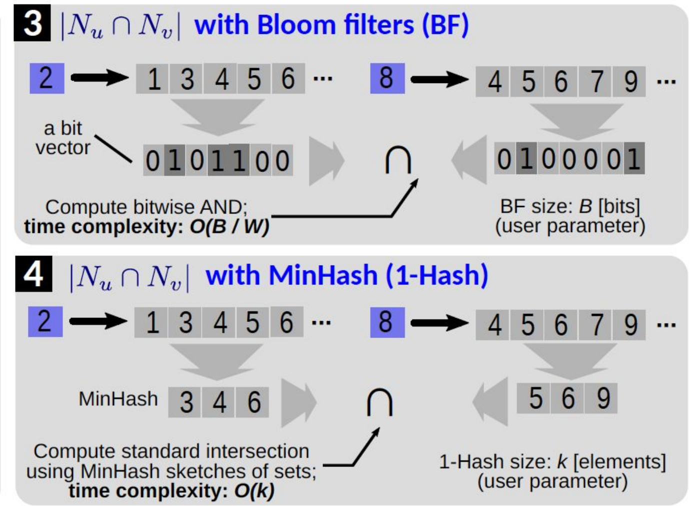
MinHash

- Take k hashes, h_1, h_2 ,.... h_k
- Compute hash for each element
- Keep values that produce the smallest per hash values
- Variant (1-Hash): keep k smallest hash values using 1 hash function

$\{\min \{h_1\}, \min \{h_2\}, \ldots, \min \{h_k\}\}$ or $\{\min \{h\}, \min \{h\} / \min \{h\}, \ldots \}$

Approximating Intersections

- Two Options:
  - Take bitwise and of Bloom Filter and compute popcount
  - Find intersection of smaller MinHash sets



Size of bloom filter (B), cache word size (W), size of MinHash set (k)

ProbGraph Implementation

- Set a storage limit as a percentage of the graph size
- Now Bloom Filter and MinHash representations exist for the neighborsets of every node with parameters chosen not to exceed this size limit.
- Choose what approximate algorithm you would like to use.
- Very fast to compute as both approximations are much smaller than original neighbor sets.
- Additionally BF is easily vectorized.

```
//Input: Graph G, two vertices u and v
//Create a standard CSR graph with G as the input graph
CSRGraph g = CSRGraph(G);
//Create a ProbGraph representation of G based on Bloom filters
ProbGraph pg = ProbGraph(g, BF, 0.25); //Use the 25% storage budget

//Derive the exact intersection cardinality |N_u ∩ N_v|
int interEX = pg.int_card(g.N(u), g.N(v));
//Derive the estimator |N_u ∩ N_v|^_AND
int interBF = pg.int_BF_AND(pg.N(u), pg.N(v));

//Compute the exact Jaccard coefficient between u and v
double jacEX = interEX / (g.N(u).size() + g.N(v).size() - interEX)
//Compute the approximate Jaccard coefficient based on BF
double jacBF = interBF / (g.N(u).size() + g.N(v).size() - interPG)
```

Listing 5: Obtaining exact and approximate Jaccard (see Listing 3)

Questions ?