Cache Oblivious Stencil Computations (2005)

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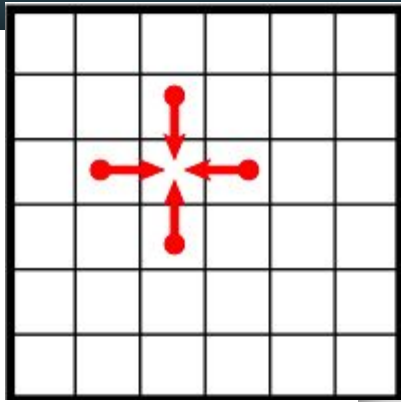
What is a stencil?



- A computation that updates elements of an array or grid according to some fixed pattern
  - The pattern is described as a **computational kernel**
- The **stencil** defines what an element is at time *t* as a function of other elements at time *t - 1,...,t - k*
- Can be applied to any *n*-dimensional grid
- When we include a time dimension, we have a *(n+1)*-dimensional spacetime grid
  - Every spacetime point, except for initial and boundary values, are computed by the computational kernel

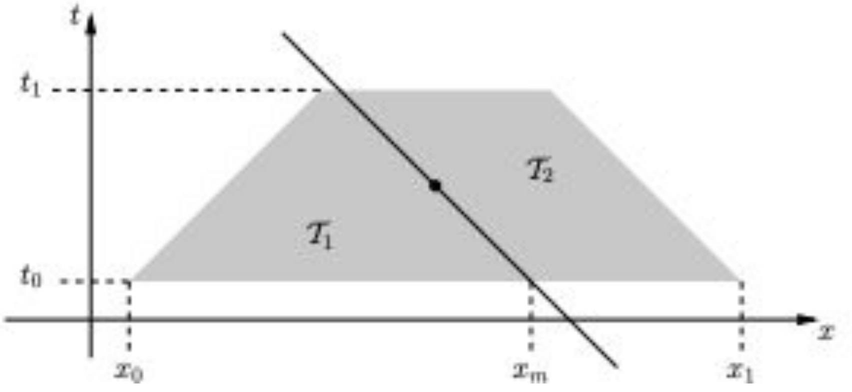
Stencil computations and Caches

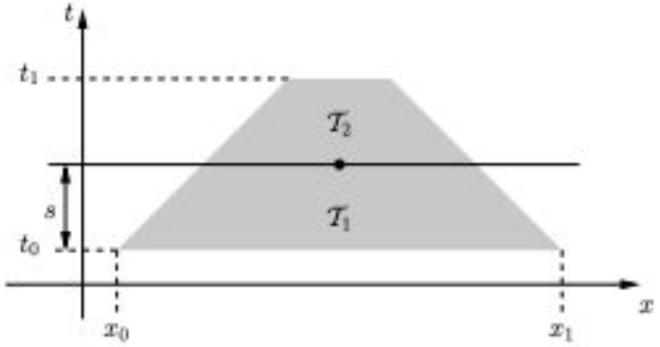
- A **stencil computation** is any traversal of spacetime that respects data dependencies of the stencil
- Simplest computes all points at *t* before any of *t+1*
- If |p| > cache size, cache misses proportional to |p|
- Storing a bounded number of time steps per space point is usually sufficient, rather than the entire spacetime
- This idea used create a cache-oblivious algorithm
- Cache-oblivious means the algorithm does not know cache size but uses cache optimally

What's so special?
Well, the fact that it's not

One-dimensional Stencil Algorithm

- 3-point stencil
- Base Case: if height == 1
  - Call kernel on all points
- If width > 2 * height *σ :
  - Space cut
    - Cut ensures each subproblem is a valid, non-empty trapezoid
- Else:
  - Time cut
- The traversal is valid because no points in T_1 depends on T_2, so it follows stencil dependencies!





Multi-dimensional Algorithm

- Allow any stencil where spacetime point $(t + 1, x)$ can be dependent on all points $(t, x + k)$ where $|k| \leq \sigma$
- Arbitrary-dimensional space time
- Idea: "Informally, for each dimension *i*, the projection of the multi-dimensional trapezoid onto the $(t, x^{(i)})$ plane looks like the 1-dimensional trapezoid"
- Perform space cuts in any dimension that allows it; time cut otherwise

Cache Complexity

- We assume that the kernel operates "in-place", the cache is "ideal" and the trapezoid is larger than the cache
  - "In-place" kernels are very common
  - Fully associative, optimal replacement, cache of two level memory system
- Lemma 1: For trapezoid *T*, let *m* be the minimum width in any dimension divided by 2. There are $O((1+n)Vol(T)/m)$ points on the surface
- Theorem 2: On an ideal cache of size *Z* and a "large" trapezoid, the algorithm incurs $O(Vol(T)/Z^{1/n})$ cache misses, as opposed to $O(Vol(T))$.
  - When a subproblem *S* gets small enough, it incurs $O(Vol(S))$ cache misses
  - Because of how we divide to get *S*, there are bounds on the height of *S* which allow for $O(Vol(S)/Z^{1/n})$ cache misses per subproblem

Questions and Comments?