Engineering a Cache-Oblivious Sorting Algorithm (Brodal et al.)

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Motivation

- Sorting is a fundamental problem.
- Cache obliviousness provides guarantees regardless of cache spec.
  - No need to fine tuning.
  - No need for parameter dependence.

Set up

- Working in the RAM model, assuming input sizes that fit in RAM.
- Target runtime (in cache misses): $O(N/B \log_{M} (N))$
  - M is size of cache
  - B is size of the cache line
  - N is size of input
- Need the (weaker) tall cache assumption: $M > B^{1+c}$, $c > 0$ (Brodal et al)
  - Pays additional cost factor of $1/c$
  - Standard tall cache assumption: $M > B^2$

Algorithm

Main algorithm:

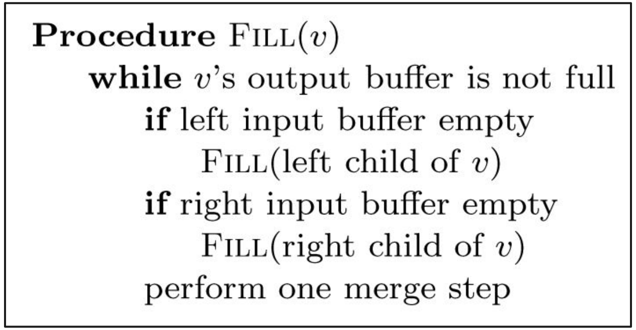
1. **Split** N into N^(1/d) segments of size N^(1-1/d) each
2. **Sort** each segment **recursively** (use standard sort for base case)
3. Apply a **(N^(1/d))-merger** on the sorted segments.

K-merger algorithm:

1. **Construct** a k-merger tree (16-merger tree shown in figure)
    a. With carefully constructed **buffer sizes**
2. Apply the ***fill*** procedure enough times to sort
    a. **Each invocation sorts k^d** elements.

Buffer sizes of a k-merger:

1. Buffer size **at the middle (depth d/2)** of the tree are: a*d^(3/2)
2. Recurse on top and bottom trees.
    a. 'Van Emde Boas'-style recursion



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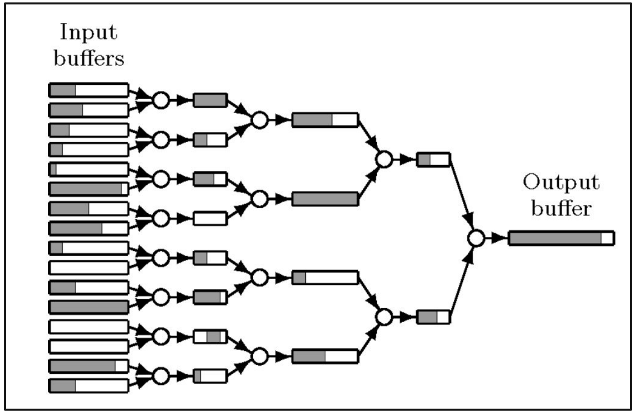
Procedure FILL($v$)
    while $v$'s output buffer is not full
        if left input buffer empty
            FILL(left child of $v$)
        if right input buffer empty
            FILL(right child of $v$)
        perform one merge step
```



Analysis:

High level idea:

- Sorting is constrained within one subset of the buffers at a time due to the Van Emde Boas-style recursion of setting buffer sizes.
- See board for intuition

Implementation and Experiments

- Machines used for evaluation:
  - Pentium 4, Pentium III, MIPS 10000, AMD Athlon, Itanium 2
- Merger implementation
  - Recursive vs iterative
- Memory navigation:
  - Pointer based, index arithmetic
- Memory layout:
  - BFS, DFS, Van Emde Boas.
  - Lay out nodes and buffers separately, vs together
- Memory allocation:
  - Custom allocator, standard allocator (only used with pointer-based navigation)
- **Results of 28 experiments: best choice of parameters:**
  - **(1) recursive invocation, (2) pointer-based navigation, (3) vEB layout**
  - **(4) nodes and buffers laid out separately, and (5) allocation by the standard allocator.**

More parameters!

- Varying degree of the merger: z = {2..9}
  - **Best choice: 4 or 5**
- Merger construction caching
  - Gave speedup of 3-5%
- Buffer size scaling parameter ***a***, and ***d***.
  - Best choice for ***a*** = 16, and ***d*** = 2
- Base-case sorting algorithm
  - Use std::sort

Comparisons and baselines

- Compared against cache aware sorting algorithms as well as quicksort.
- See paper for charts!
- Main takeaway: performance depends on the architecture and the input size
    - In some cases the overhead of funnelsort is not worth the gain
    - For architectures with fast CPUs (where cache misses are costlier in comparison) and large input sizes, Funnelsort wins!

Discussion question

- Can we have an even simpler algorithm?