An Experimental Analysis of a Compact Graph Representation

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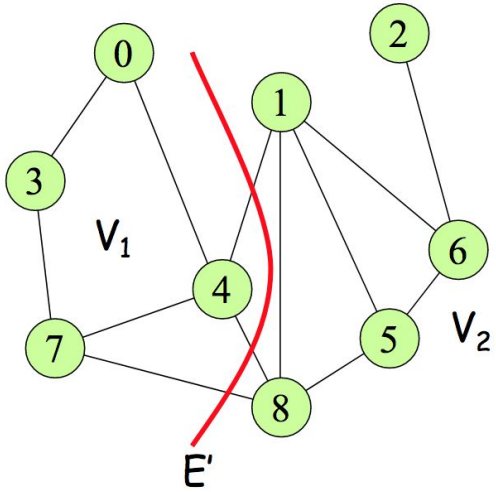
Motivation

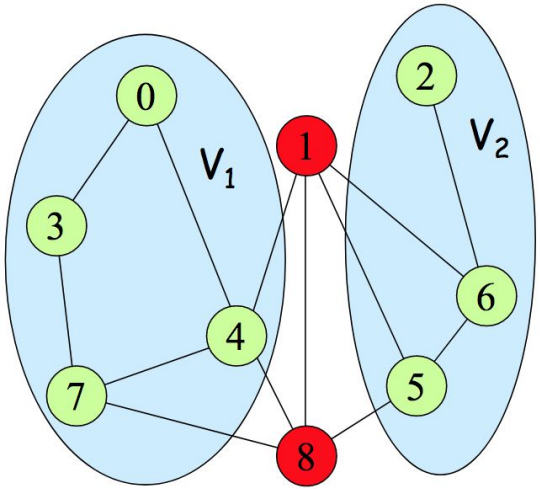
- Graphs are too big to fit in memory
  - Even medium-sized graphs on devices with limited memory
- Compression helps a lot for performance too - locality!

- This paper builds upon a past paper where they first introduce the graph separator-based representation

Graph Separators

- Edge separator = a set of edges that, when removed, partitions the graph into two almost equal sized parts
- Vertex separator = a set of vertices that, when removed, partitions the graph into two almost equal parts



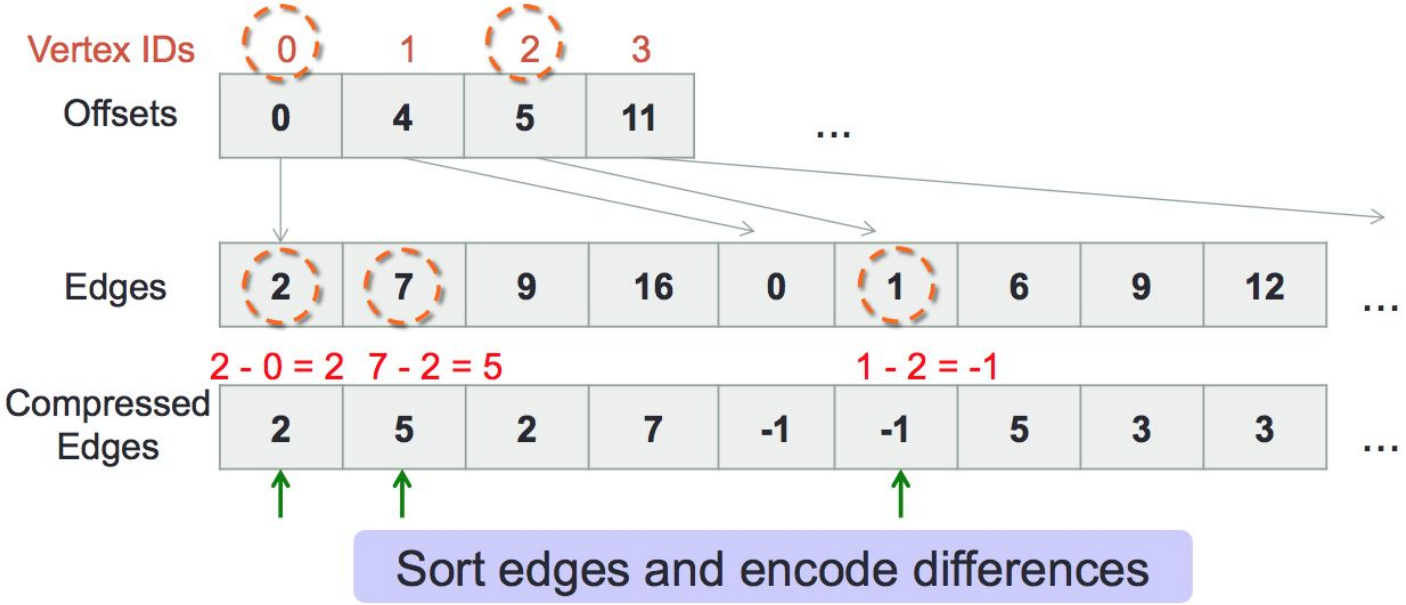


Graph Separators

- A graph has good separators if it and its subgraphs have minimum separators that are significantly better than expected for a random graph of its size
  - Means the graph has good locality
- Real-world graphs have good separators
  - Real-world graphs are based on communities
  - Locality is super important!!!

But why do we care about graph separators?

Graph representation



- Graph reordering to improve locality
  - Goal: give neighbors IDs close to vertex ID
  - BFS, DFS, METIS, our own separator-based algorithm

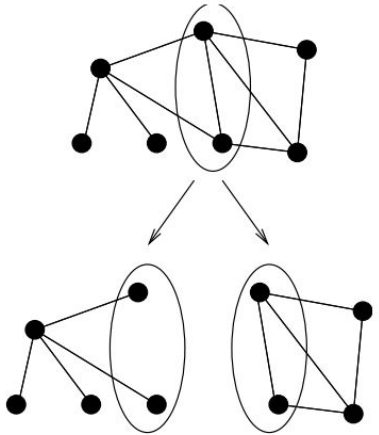
Encoding with Graph Separators

- Assume that we have a graph separator algorithm that returns a separator
- Given a graph G, construct a separator tree
  - Each node of the tree contains a subgraph of G and a separator for that subgraph
  - The children of a node contain the two components of the graph induced by the separator
  - The leaves are single nodes

```
BuildTree(V, E)
  if |E| = 1 then
    return V

  (V_a, V_sep, V_b) ← FindSeparator(V, E)
  E_a ← {(u, v) ∈ E | u ∈ V_a ∨ v ∈ V_a}
  E_b ← E − E_a
  V_{a,sep} ← V_a ∪ V_sep
  V_{b,sep} ← V_b ∪ V_sep

  T_a ← BuildTree(V_{a,sep}, E_a)
  T_b ← BuildTree(V_{b,sep}, E_b)
  return SeparatorTree(T_a, V_sep, T_b)
```



Encoding with Graph Separators

Our compression algorithm works as follows:

- Generate an edge separator tree for the graph
- Label the vertices in-order across the leaves
- Use an adjacency table to represent the relabeled

Implementation - Separator Trees

- “Bottom-up” separator algorithm with child-flipping
- Begins with complete graph and repeatedly collapses edges until a single vertex remains
  - Based on the priority metric $w(E_{AB}) / s(A)\, s(B)$
- “Child-flipping” - when we construct the tree, choose which side is the left and which side is the right in a way to maximize locality

Implementation - Indexing

- Semi-direct-16 stores the start locations for sixteen vertices in five 32-bit words
  - Word 1 contains start location of Vertex 0
  - Word 2 contains three ten-bit offsets from Vertex 0 to Vertices 4, 8, 12
  - Words 3-5 contain twelve eight-bit offsets from one of these four vertices to the remaining vertices

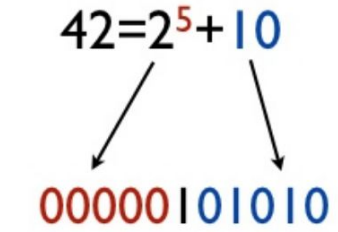
Implementation - Codes and Decoding

- Gamma codes - store an integer d by using a unary code for log(d) followed by a binary code for its offset
- Snip, Nibble, and Byte codes

(Turns out Byte codes are the fastest)

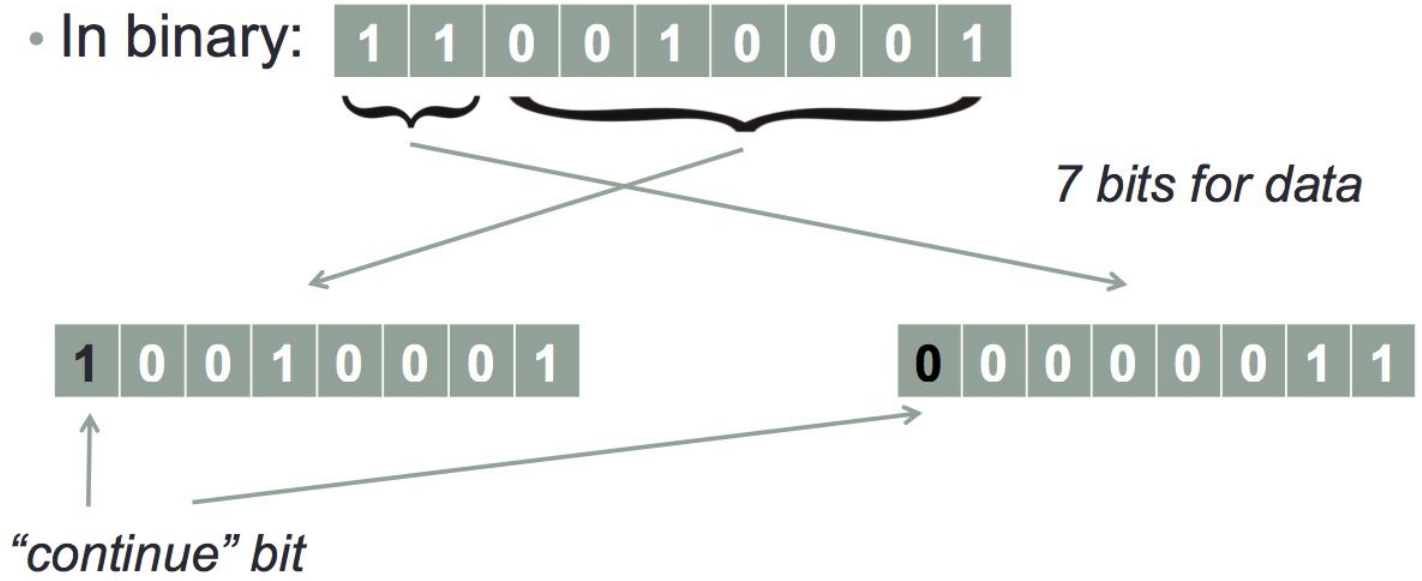
number	2^n	output
1	2^0+0	1
2	2^1+0	010
3	2^1+1	011
4	2^2+0	00100
5	2^2+1	00101
6	2^2+2	00110
7	2^2+3	00111
8	2^3+0	0001000
9	2^3+1	0001001
10	2^3+2	0001010
11	2^3+3	0001011
12	2^3+4	0001100
13	2^3+5	0001101
14	2^3+6	0001110
15	2^3+7	0001111
16	2^4+0	000010000
17	2^4+1	000010001

Example



Variable-length codes

- k-bit codes
  - Encode value in chunks of k bits
  - Use k-1 bits for data, and 1 bit as the “continue” bit
- Example: encode “401” using 8-bit (byte) code
- In binary:



Dynamic Representation

- Incremental insertions + deletions of edges
- Size for a vertex can change, so need to dynamically assign memory
- Fixed block size memory allocation
  - Data structure initially has an array with one memory block for each vertex
  - If memory runs out, the vertex is assigned additional blocks from a pool of spare memory blocks
- Blocks for a single vertex are stored via a linked list
  - Each block contains an 8-bit nonce i
  - hash(current_address, i) maps to the address of the next block in the linked list
- To ensure memory locality, a separate pool of contiguous memory blocks is allocated for each 1024 vertices of the graph

Dynamic Representation - Caching

- Bad to repeatedly encode + decode neighbors
- When a vertex is queried, its neighbors are decoded and stored in a temporary LRU-based cache
- A modified vertex that is flushed from the cache is written back to the main data structure in compressed form

Experimental Results

- DFS - visits every edge once in a non-trivial order
- Reading edges - accessing vertices in linear / random order
- Inserting edges - linear, transpose, or random
- Compared to other (non-compressed) methods
  - Adjacency lists - neighbors of a vertex are stored in singly linked-list format
  - Adjacency array - adjacency lists in array format
  - The ordering of vertices matters a lot! Using the separator-based ordering improved performance by a factor of up to 7

Graph	Vtxs	Edges	Max Degree	Source
auto	448695	6629222	37	3D mesh [35]
feocean	143437	819186	6	3D mesh [35]
m14b	214765	3358036	40	3D mesh [35]
ibm17	185495	4471432	150	circuit [1]
ibm18	210613	4443720	173	circuit [1]
CA	1971281	5533214	12	street map [34]
PA	1090920	3083796	9	street map [34]
googleI	916428	5105039	6326	web links [10]
googleO	916428	5105039	456	web links [10]
lucent	112969	363278	423	routers [25]
scan	228298	640336	1937	routers [25]

Table 1: Properties of the graphs used in our experiments.

Static Algorithm (compared to Adjacency Array)

	Array			Our Structure									
	Rand	Sep		Byte		Nibble		Snip		Gamma		DiffByte	
Graph	T_1	T/T_1	Space	T/T_1	Space	T/T_1	Space	T/T_1	Space	T/T_1	Space	T/T_1	Space
auto	0.268s	0.313	34.17	0.294	10.25	0.585	7.42	0.776	6.99	1.063	7.18	0.399	12.33
feocean	0.048s	0.312	37.60	0.312	12.79	0.604	10.86	0.791	11.12	1.0	11.97	0.374	13.28
m14b	0.103s	0.388	34.05	0.349	10.01	0.728	7.10	0.970	6.55	1.320	6.68	0.504	11.97
ibm17	0.095s	0.536	33.33	0.536	10.19	1.115	7.72	1.400	7.58	1.968	7.70	0.747	12.85
ibm18	0.113s	0.398	33.52	0.442	10.24	0.867	7.53	1.070	7.18	1.469	7.17	0.548	12.16
CA	0.920s	0.126	43.40	0.146	14.77	0.243	10.65	0.293	10.55	0.333	11.25	0.167	14.81
PA	0.487s	0.137	43.32	0.156	14.76	0.258	10.65	0.310	10.60	0.355	11.28	0.178	14.80
lucent	0.030s	0.266	41.95	0.3	14.53	0.5	11.05	0.566	10.79	0.700	11.48	0.333	14.96
scan	0.067s	0.208	43.41	0.253	15.46	0.402	11.84	0.477	11.61	0.552	12.14	0.298	16.46
googleI	0.367s	0.226	37.74	0.258	11.93	0.405	8.39	0.452	7.37	0.539	7.19	0.302	13.39
googleO	0.363s	0.250	37.74	0.278	12.59	0.460	9.72	0.556	9.43	0.702	9.63	0.327	13.28
Avg		0.287	38.202	0.302	12.501	0.561	9.357	0.696	9.07	0.909	9.424	0.380	13.662

Table 2: Performance of our **static** algorithms compared to performance of an adjacency array representation. Space is in bits per edge; time is for a DFS, normalized to the first column, which is given in seconds.

Dynamic Algorithm

	3		4		8		12		16		20	
Graph	T_1	Space	T/T_1	Space	T/T_1	Space	T/T_1	Space	T/T_1	Space	T/T_1	Space
auto	0.318s	11.60	0.874	10.51	0.723	9.86	0.613	10.36	0.540	9.35	0.534	11.07
feocean	0.044s	14.66	0.863	13.79	0.704	12.97	0.681	17.25	0.727	22.94	0.750	28.63
m14b	0.146s	11.11	0.876	10.07	0.684	9.41	0.630	10.00	0.554	8.92	0.554	10.46
ibm17	0.285s	12.95	0.849	11.59	0.614	10.44	0.529	10.53	0.491	10.95	0.459	11.39
ibm18	0.236s	12.41	0.847	11.14	0.635	10.12	0.563	10.36	0.521	10.97	0.5	11.64
CA	0.212s	10.62	0.943	12.42	0.952	23.52	1.0	35.10	1.018	46.68	1.066	58.26
PA	0.119s	10.69	0.941	12.41	0.949	23.35	1.0	34.85	1.025	46.35	1.058	57.85
lucent	0.018s	13.67	0.888	14.79	0.833	22.55	0.833	31.64	0.833	41.22	0.888	51.09
scan	0.034s	15.23	0.941	16.86	0.852	26.39	0.852	37.06	0.852	48.08	0.882	59.34
googleI	0.230s	11.91	0.895	12.04	0.752	15.71	0.730	20.53	0.730	25.78	0.726	31.21
googleO	0.278s	13.62	0.863	13.28	0.694	15.65	0.658	19.52	0.640	24.24	0.676	29.66
Avg		12.58	0.889	12.62	0.763	16.36	0.735	21.56	0.721	26.86	0.736	32.78

Table 3: Performance of our dynamic algorithm using nibble codes with various block sizes. For each size we give the space needed in bits per edge (assuming enough blocks to leave the secondary hash table 80% full) and the time needed to perform a DFS. Times are normalized to the first column, which is given in seconds. .

Dynamic Algorithm (compared to linked lists)

	Linked List							Our Structure					
	Random Vtx Order			Sep Vtx Order				Space Opt			Time Opt		
Graph	Rand T_1	Trans T/T_1	Lin T/T_1	Rand T/T_1	Trans T/T_1	Lin T/T_1	Space	Block Size	Time T/T_1	Space	Block Size	Time T/T_1	Space
auto	1.160s	0.512	0.260	0.862	0.196	0.093	68.33	16	0.148	9.35	20	0.087	13.31
feocean	0.136s	0.617	0.389	0.801	0.176	0.147	75.21	8	0.227	12.97	10	0.117	14.71
m14b	0.565s	0.442	0.215	0.884	0.184	0.090	68.09	16	0.143	8.92	20	0.086	13.53
ibm17	0.735s	0.571	0.152	0.904	0.357	0.091	66.66	12	0.205	10.53	20	0.118	14.52
ibm18	0.730s	0.524	0.179	0.890	0.276	0.080	67.03	10	0.190	10.13	20	0.108	14.97
CA	1.240s	0.770	0.705	0.616	0.107	0.101	86.80	3	0.170	10.62	5	0.108	15.65
PA	0.660s	0.780	0.701	0.625	0.112	0.109	86.64	3	0.180	10.69	5	0.115	15.64
lucent	0.063s	0.634	0.492	0.730	0.190	0.142	83.90	3	0.285	13.67	6	0.174	20.49
scan	0.117s	0.735	0.555	0.700	0.188	0.128	86.82	3	0.290	15.23	8	0.170	28.19
googleI	0.975s	0.615	0.376	0.774	0.164	0.096	75.49	4	0.211	12.04	16	0.125	28.78
googleO	0.960s	0.651	0.398	0.786	0.162	0.108	75.49	5	0.231	13.54	16	0.123	26.61
Avg		0.623	0.402	0.779	0.192	0.108	76.405		0.207	11.608		0.121	18.763

Table 4: The performance of our **dynamic** algorithms compared to linked lists. For each graph we give the space- and time-optimal block size. Space is in bits per edge; time is for a DFS, normalized to the first column, which is given in seconds.

Machines

- Pentium 4 is more powerful, larger cache-size, supports quadruple loads + hardware prefetching
  - Much better at loading consecutive blocks in memory, not good for random access

Graph	DFS	Read		Find Next	Insert			Space
		Linear	Random		Linear	Random	Transpose	
ListRand	1.000	0.099	0.744	0.121	0.571	28.274	3.589	76.405
ListOrdr	0.322	0.096	0.740	0.119	0.711	28.318	0.864	76.405
LEDARand	2.453	1.855	2.876	2.062	16.802	21.808	16.877	432.636
LEDAOrdr	1.119	0.478	2.268	0.519	7.570	20.780	7.657	432.636
DynSpace	0.633	0.440	0.933	0.324	14.666	23.901	15.538	11.608
DynTime	0.367	0.233	0.650	0.222	9.725	15.607	10.183	18.763
CachedSpace	0.622	0.431	0.935	0.324	2.433	28.660	8.975	13.34
CachedTime	0.368	0.240	0.690	0.246	2.234	19.849	6.600	19.073
ArrayRand	0.945	0.095	0.638	0.092	—	—	—	38.202
ArrayOrdr	0.263	0.092	0.641	0.092	—	—	—	38.202
Byte	0.279	0.197	0.693	0.205	—	—	—	12.501
Nibble	0.513	0.399	0.873	0.340	—	—	—	9.357
Snip	0.635	0.562	1.044	0.447	—	—	—	9.07
Gamma	0.825	0.710	1.188	0.521	—	—	—	9.424

Table 5: Summary of space and normalized times for various operations on the Pentium 4.

Graph	DFS	Read		Find Next	Insert			Space
		Linear	Random		Linear	Random	Transpose	
ListRand	1.000	0.631	0.995	0.508	1.609	17.719	3.391	76.405
ListOrdr	0.710	0.626	0.977	0.516	1.551	17.837	1.632	76.405
LEDARand	3.163	2.649	3.038	2.518	17.543	19.342	17.880	432.636
LEDAOrdr	2.751	2.168	2.878	1.726	11.846	19.365	11.783	432.636
DynSpace	0.626	0.503	0.715	0.433	17.791	22.520	18.423	11.608
DynTime	0.422	0.342	0.531	0.335	13.415	16.926	13.866	17.900
CachedSpace	0.614	0.498	0.723	0.429	2.616	25.380	7.788	13.36
CachedTime	0.430	0.355	0.558	0.360	2.597	20.601	6.569	17.150
ArrayRand	0.729	0.319	0.643	0.298	—	—	—	38.202
ArrayOrdr	0.429	0.319	0.639	0.302	—	—	—	38.202
Byte	0.330	0.262	0.501	0.280	—	—	—	12.501
Nibble	0.488	0.411	0.646	0.387	—	—	—	9.357
Snip	0.684	0.625	0.856	0.538	—	—	—	9.07
Gamma	0.854	0.764	1.016	0.640	—	—	—	9.424

Table 6: Summary of space and normalized times for various operations on the Pentium III.

Discussion

- The simple and fast separator tree heuristic works well
  - Compression is not that sensitive to the quality of the separator
- Real-world graphs have small separators
- Compressed representations are faster than standard representations despite extra computation for decoding
  - Additional cost for decoding is small
  - Performance bottleneck seems to be accessing memory, not the bit operations
- Separator-based orderings had much better performance for adjacency lists and adjacency arrays (b/c of caching effects)
  - People need to pay more attention to ordering