A Framework for Processing Large Graphs in Shared Memory

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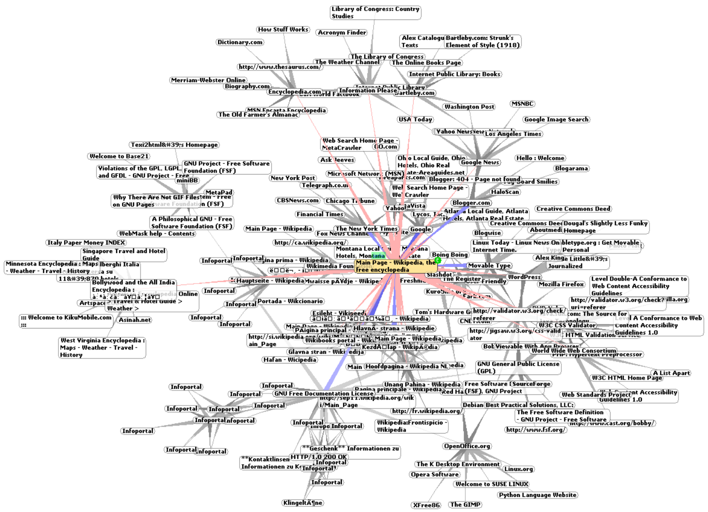
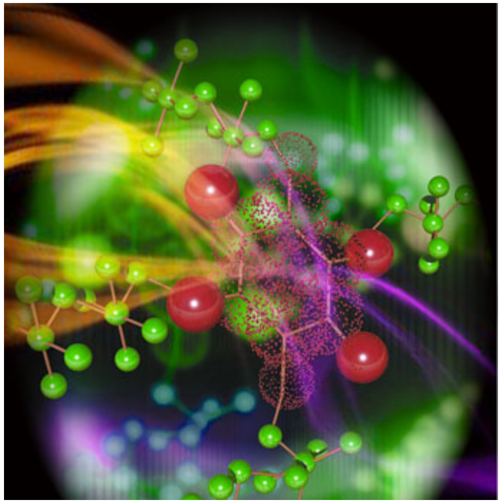
Based on joint work with Guy Blelloch and Laxman Dhulipala
(Work done at Carnegie Mellon University)

What are graphs?





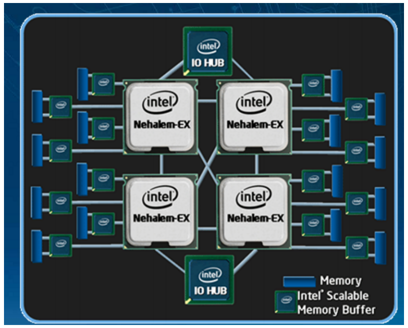
Graph Data is Everywhere!

- Can contain up to billions of vertices and edges
- Need simple, efficient, and scalable ways to analyze them

Efficient Graph Processing

- Use parallelism





- Design efficient algorithms

Breadth-first search	Single-source shortest paths
Betweenness centrality	Eccentricity estimation
Connected components	(Personalized) PageRank
…	…

  - Write/optimize code for each application
  - Build a general framework

Ligra Graph Processing Framework

EdgeMap

VertexMap

Breadth-first search
Betweenness centrality
Connected components
Triangle counting
K-core decomposition
Maximal independent set
Set cover

Single-source shortest paths
Eccentricity estimation
(Personalized) PageRank
Local graph clustering
Biconnected components
Collaborative filtering
…

Simplicity, Performance, Scalability

Graph Processing Systems

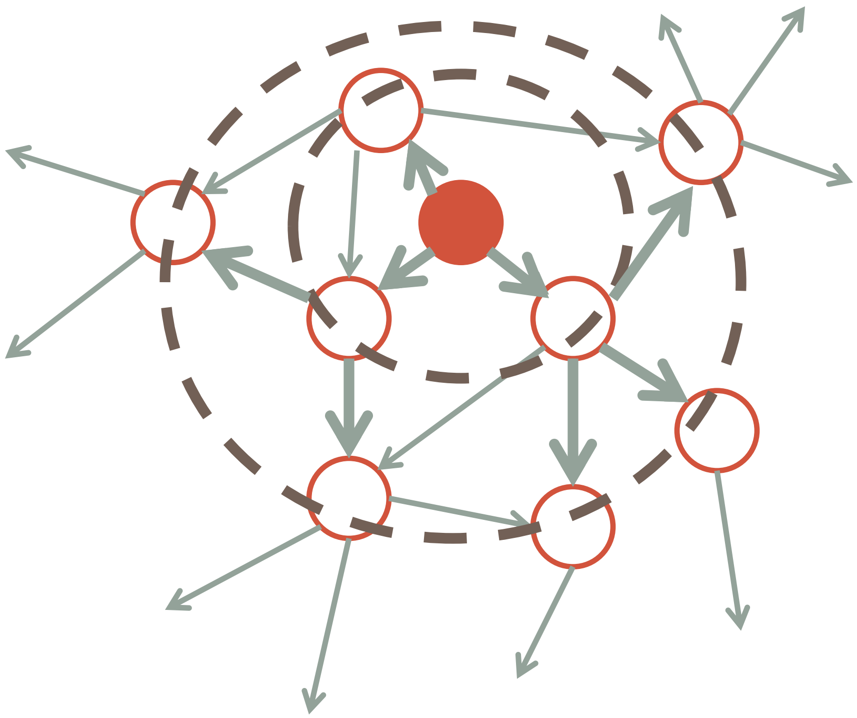
- Existing: Pregel/Giraph/GPS, GraphLab, PRISM, Pegasus, Knowledge Discovery Toolbox, GraphChi, GraphX, and many others…
- Our system: Ligra - Lightweight graph processing system for shared memory

Takes advantage of "frontier-based" nature of many algorithms (active set is dynamic and often small)

Breadth-first Search (BFS)

- Compute a BFS tree rooted at source *r* containing all vertices reachable from *r*

Applications
Betweenness centrality
Eccentricity estimation
Maximum flow
Web crawlers
Network broadcasting
Cycle detection
…



- Can process each frontier in parallel
- Race conditions, load balancing

Steps for Graph Traversal

Many graph traversal algorithms do this!

Graph

- Operate on a subset of vertices — VertexSubset
- Map computation over subset of edges in parallel
- Return new subset of vertices

} EdgeMap

- Map computation over subset of vertices in parallel — VertexMap

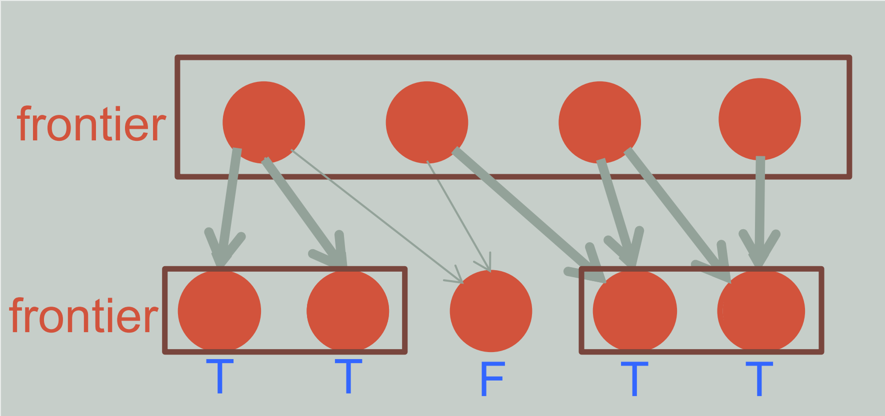
*We built the **Ligra** abstraction for these kinds of computations*

Think with flat data-parallel operators

Abstraction enables optimizations (hybrid traversal and graph compression)

Breadth-first Search in Ligra

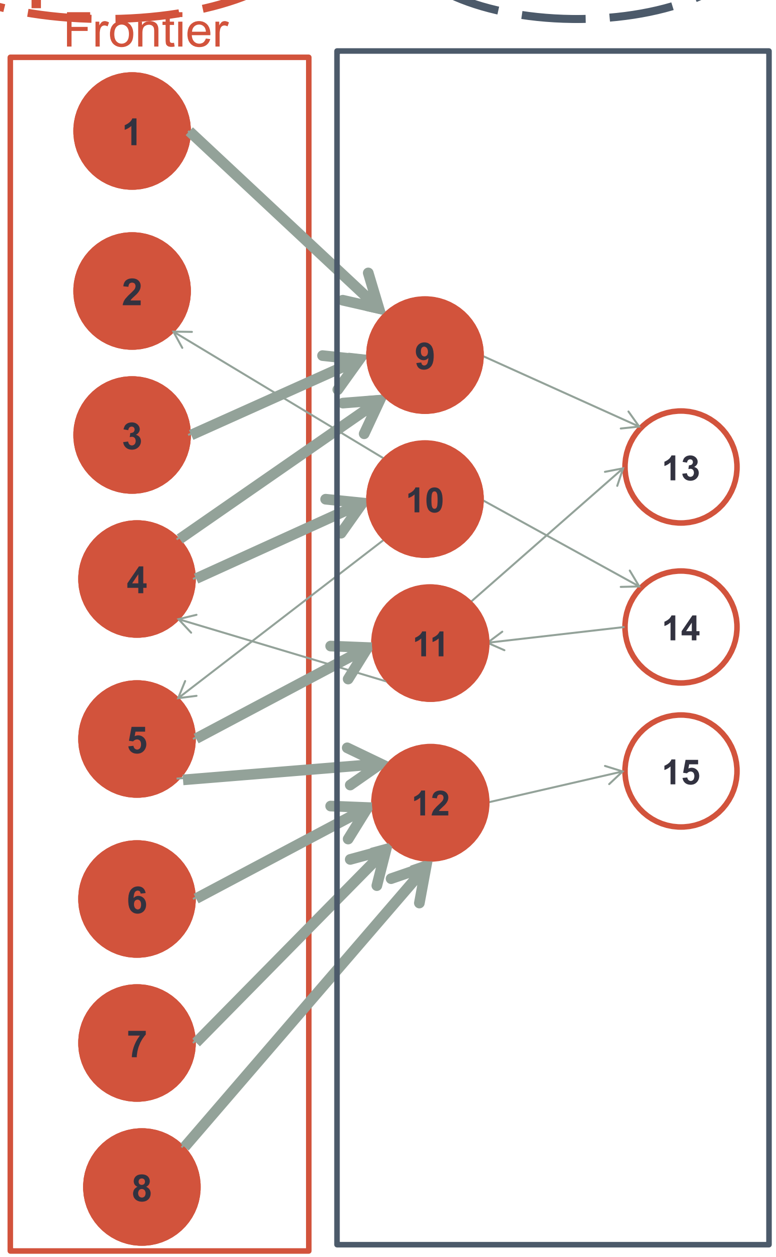
```
parents = {-1, …, -1};   //-1 indicates "unexplored"

procedure UPDATE(s, d):
    return compare_and_swap(parents[d], -1, s);

procedure COND(v):
    return parents[v] == -1;   //checks if "unexplored"

procedure BFS(G, r):
    parents[r] = r;
    frontier = {r}; //VertexSubset
    while (size(frontier) > 0):
        frontier = EDGEMAP(G, frontier, UPDATE, COND);
```



Actual BFS code in Ligra

```
#include "ligra.h"

struct BFS_F {
  intT* Parents;
  BFS_F(intT* _Parents) : Parents(_Parents) {}
  inline bool update (intT s, intT d) { //Update
    if(Parents[d] == -1) { Parents[d] = s; return 1; }
    else return 0;
  }
  inline bool updateAtomic (intT s, intT d){ //atomic version of Update
    return (CAS(&Parents[d],(intT)-1,s));
  }
  //cond function checks if vertex has been visited yet
  inline bool cond (intT d) { return (Parents[d] == -1); }
};

template <class vertex>
void Compute(graph<vertex> GA, intT start) {
  intT n = GA.n;
  //creates Parents array, initialized to all -1, except for start
  intT* Parents = newA(intT,GA.n);
  parallel_for(intT i=0;i<GA.n;i++) Parents[i] = -1;
  Parents[start] = start;

  vertexSubset Frontier(n,start); //creates initial frontier

  while(!Frontier.isEmpty()){ //loop until frontier is empty
    vertexSubset output = edgeMap(GA, Frontier, BFS_F(Parents));
    Frontier.del();
    Frontier = output; //set new frontier
  }
  Frontier.del();
  free(Parents);
}
```

Sparse or Dense EdgeMap?



- Dense method better when frontier is large and many vertices have been visited
- Sparse (traditional) method better for small frontiers
- Switch between the two methods based on frontier size [Beamer et al. SC '12]

Limited to BFS?

EdgeMap

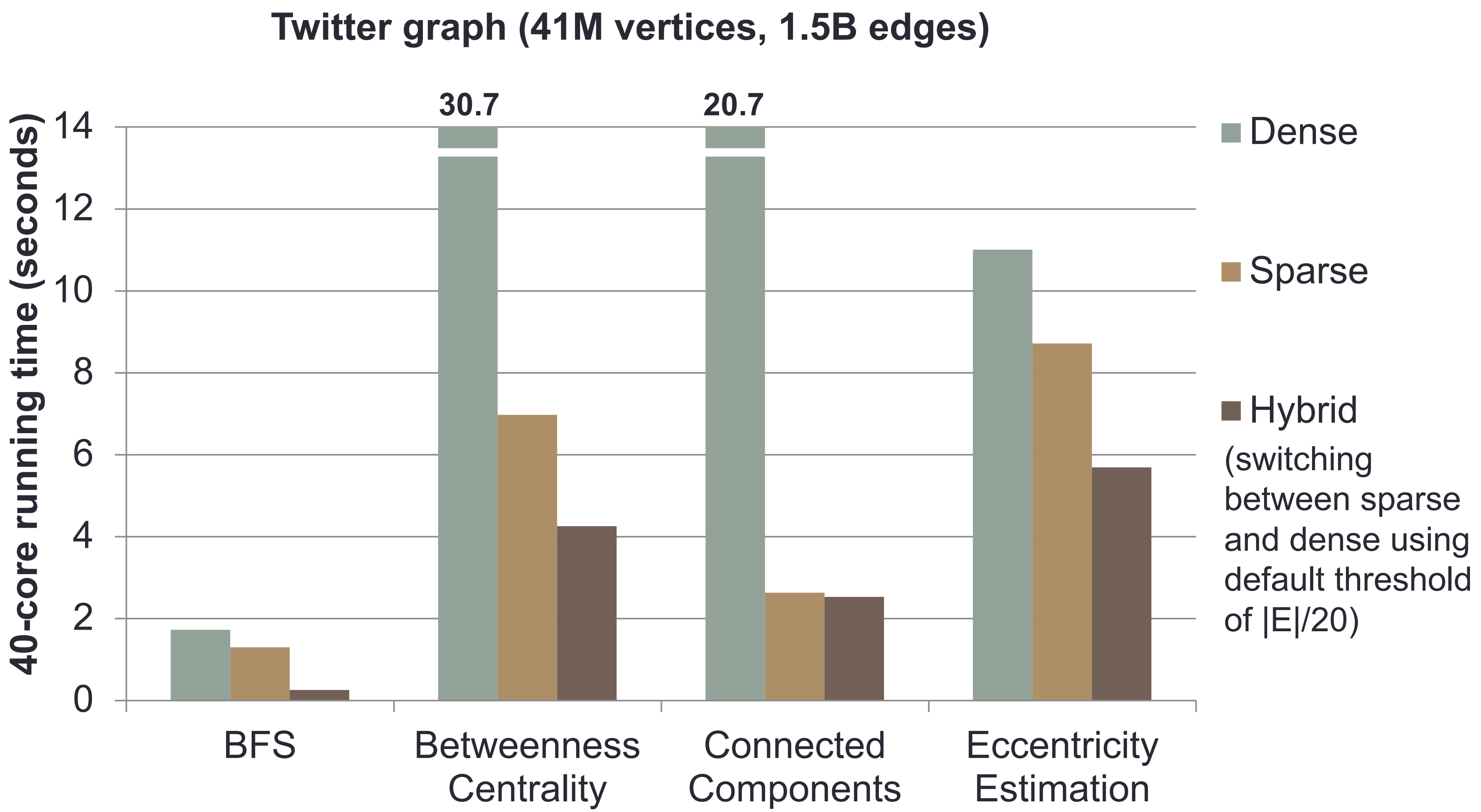
```
procedure EDGEMAP(G, frontier, Update, Cond):
    if (size(frontier) + sum of out-degrees > threshold) then:
        return EDGEMAP_DENSE(G, frontier, Update, Cond);
    else:
        return EDGEMAP_SPARSE(G, frontier, Update, Cond);
```

Loop through outgoing edges of frontier vertices in parallel

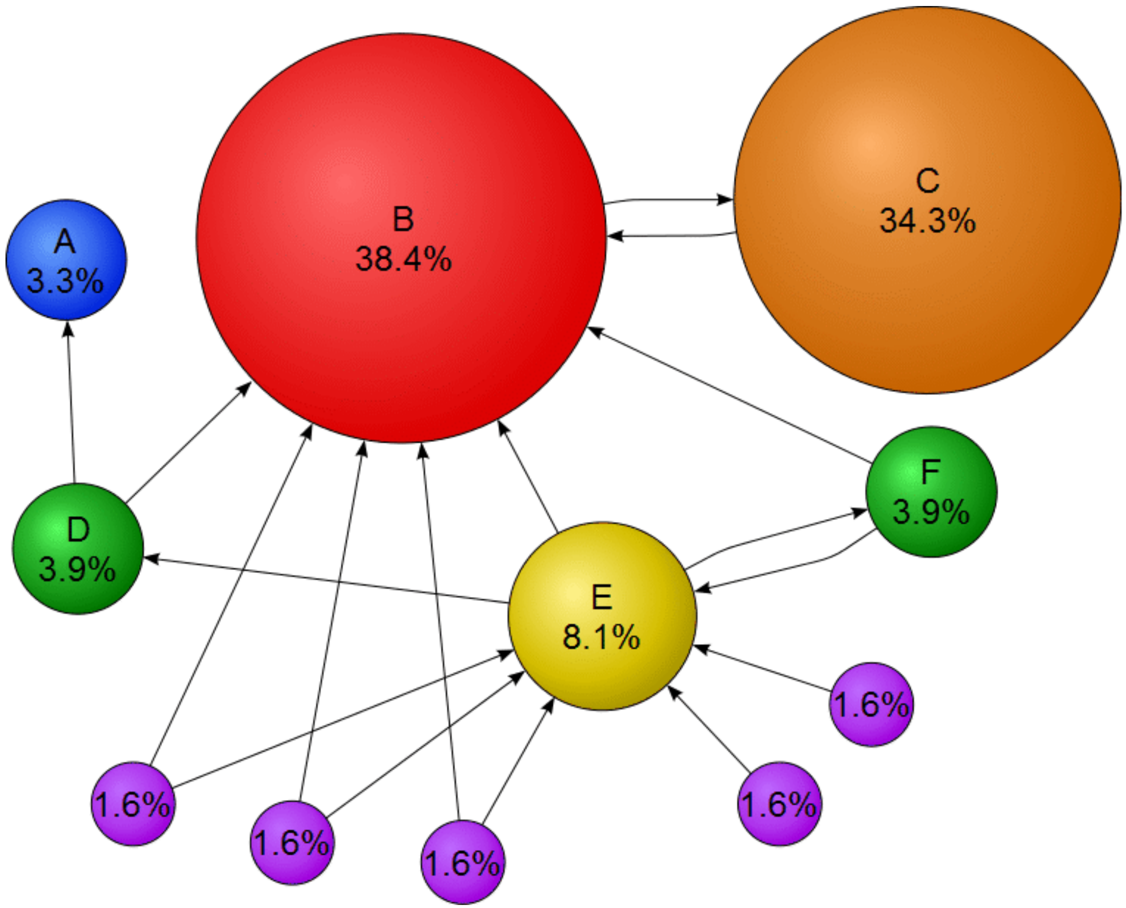
Loop through incoming edges of “unexplored” vertices (in parallel), breaking early if possible

- More general than just BFS!
- Generalized to many other problems
  - For example, betweenness centrality, connected components, sparse PageRank, shortest paths, eccentricity estimation, graph clustering, k-core decomposition, set cover, etc.
- Users need not worry about this

Frontier-based approach enables hybrid traversal

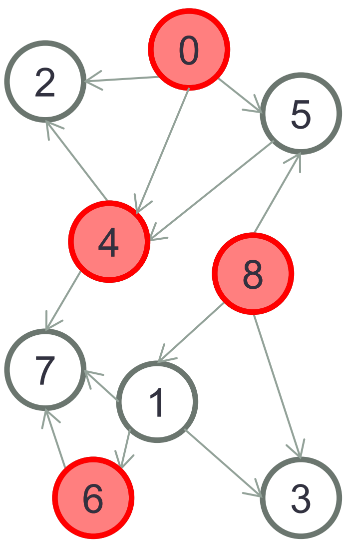


PageRank

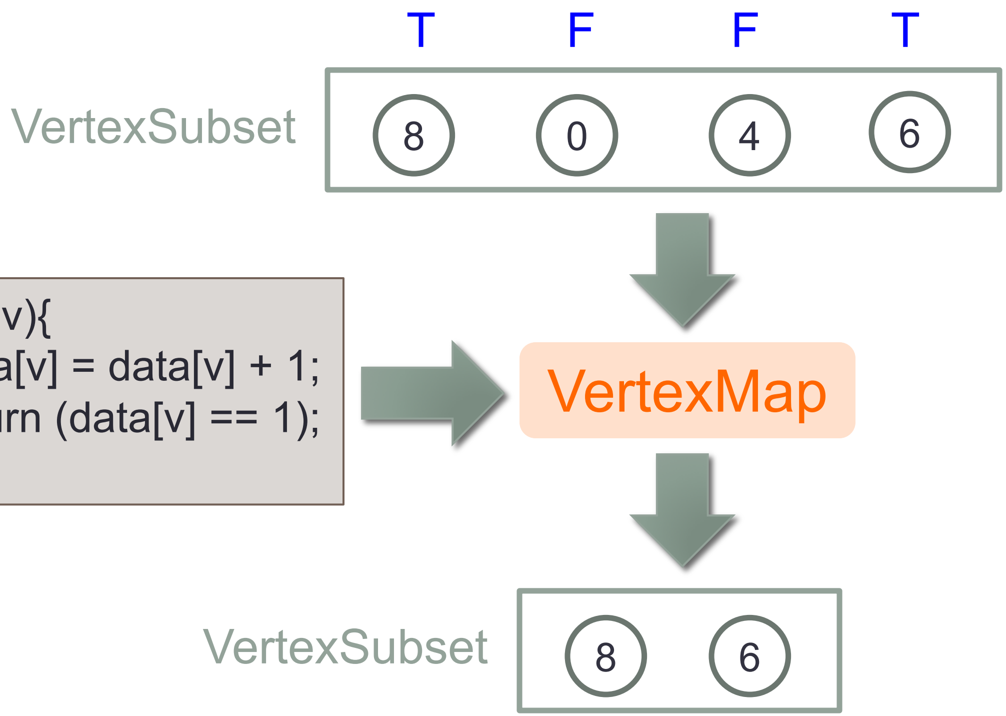


$$\mathrm{PR}[v] = \frac{1-\gamma}{|V|} + \gamma \sum_{u \in N^-(v)} \frac{\mathrm{PR}[u]}{deg^+(u)}$$

VertexMap




```
bool f(v){
    data[v] = data[v] + 1;
    return (data[v] == 1);
}
```



PageRank in Ligra

```
p_curr = {1/|V|, …, 1/|V|};      p_next = {0, …, 0};      diff = {};      error =∞;

procedure UPDATE(s, d):
    atomic_increment(p_next[d], p_curr[s] / degree(s));
    return 1;

procedure COMPUTE(i):
    p_next[i] = α · p_next[i] + (1- α) · (1/|V|);
    diff[i] = abs(p_next[i] – p_curr[i]);
    p_curr[i] = 0;
    return 1;

procedure PageRank(G, α, ε):
    frontier = {0, …, |V|-1};
    while (error > ε):
        frontier = EDGEMAP(G, frontier, UPDATE, COND_true);
        frontier = VERTEXMAP(frontier, COMPUTE);
        error = sum of diff entries;
        swap(p_curr, p_next)
    return p_curr;
```

PageRank

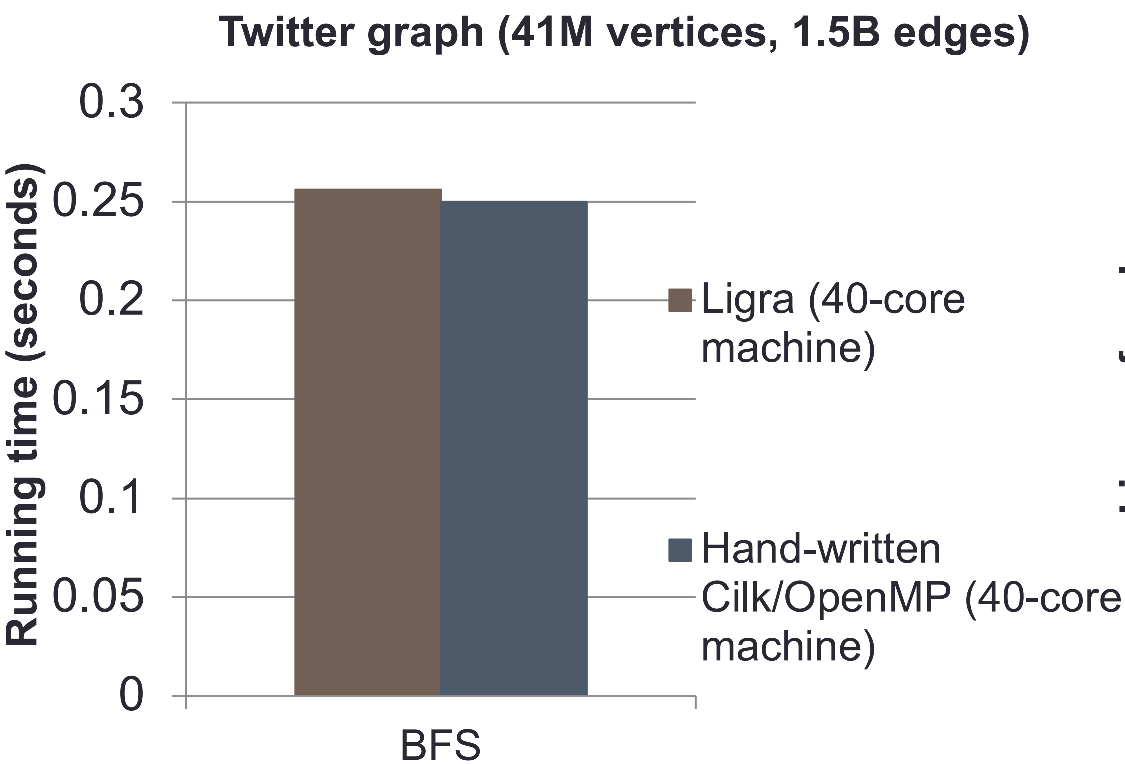
- *Sparse version?*
  - PageRank-Delta: Only update vertices whose PageRank value has changed by more than some Δ-fraction (discussed in PowerGraph and McSherry WWW '05)

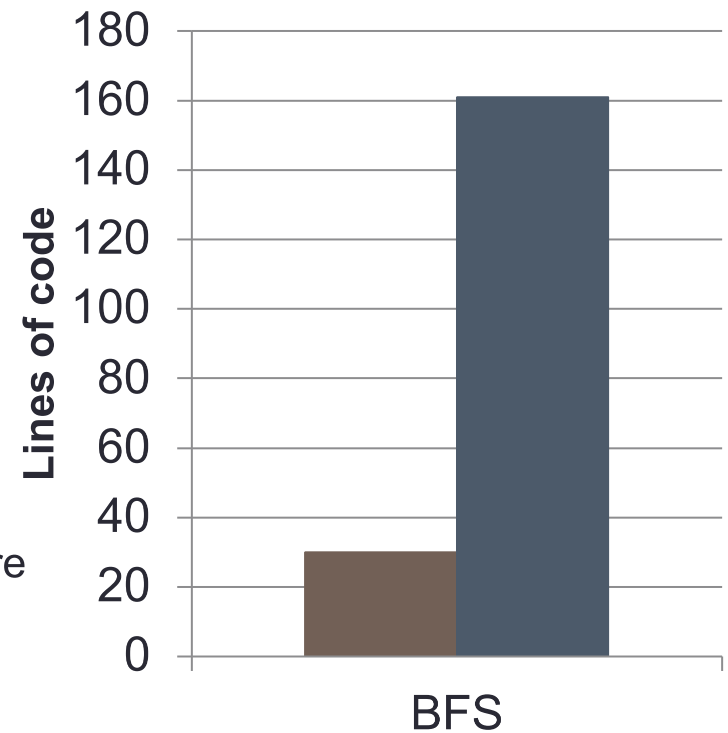
PageRank-Delta in Ligra

```
PR[i] = {1/|V|, …, 1/|V|};
nghSum = {0, …, 0};
Change = {};                    //store changes in PageRank values

procedure UPDATE(s, d):                 //passed to EdgeMap
    atomic_increment(nghSum[d], Change[s] / degree(s));
    return 1;

procedure COMPUTE(i):                   //passed to VertexMap
    Change[i] = α · nghSum[i];
    PR[i] = PR[i] + Change[i];
    return (abs(Change[i]) > Δ);    //check if absolute value of change is big enough
```

Performance of Ligra

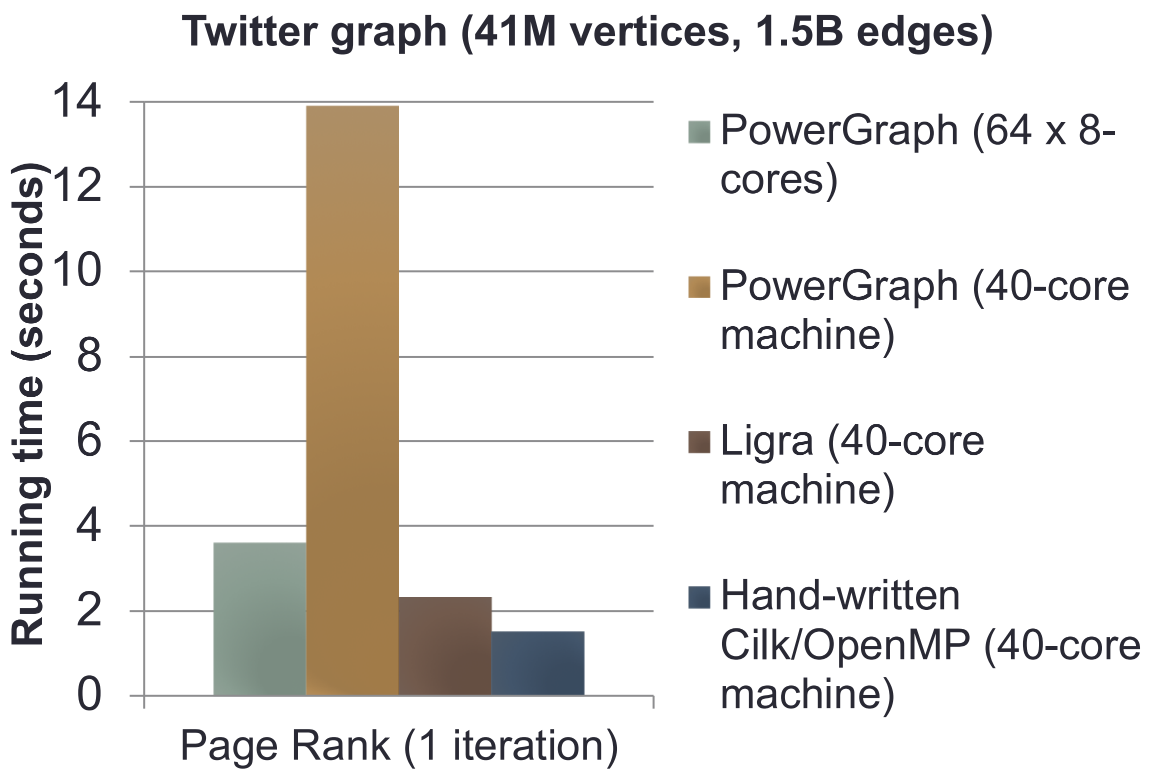
Ligra BFS Performance

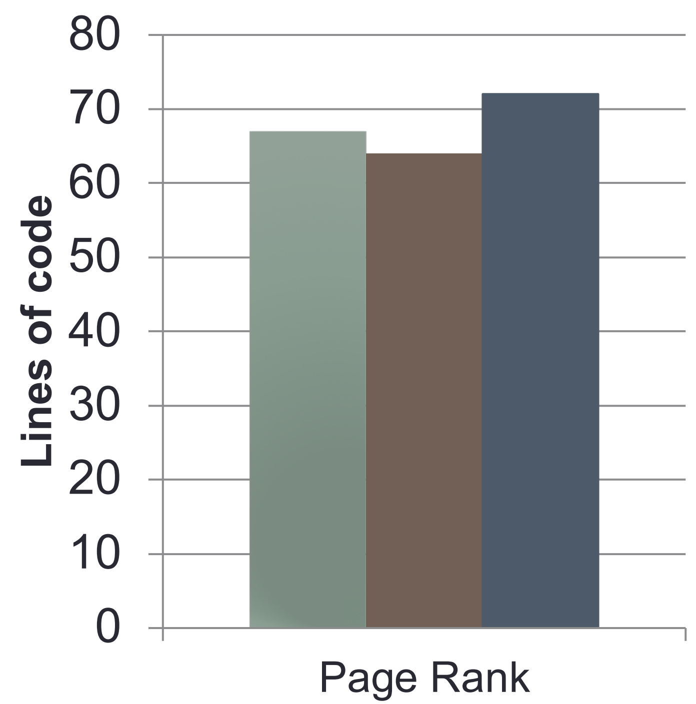




- Comparing against hybrid traversal BFS code by Beamer et al.

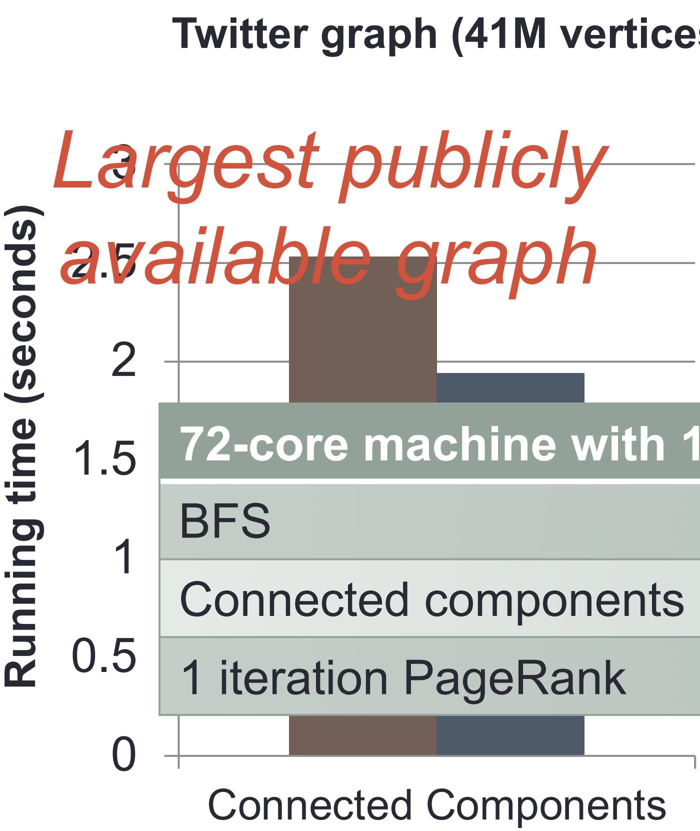
Ligra PageRank Performance





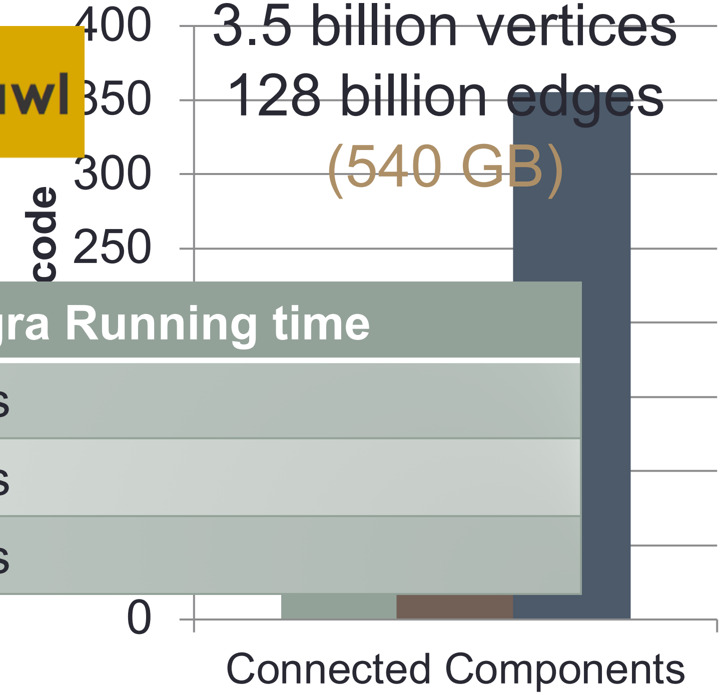
- Easy to implement "sparse" version of PageRank in Ligra

Connected Components Performance



Largest publicly available graph

Common Crawl



3.5 billion vertices
128 billion edges
(540 GB)

72-core machine with 1TB RAM	Ligra Running time
BFS	12s
Connected components	42s
1 iteration PageRank	28s

- Ligra's performance is close to hand-written code
- Faster than best existing system
- Subsequent systems have used Ligra's abstraction and hybrid traversal idea, e.g., Galois [SOSP '13], Polymer [PPoPP '15], Gunrock [PPoPP '16], Gemini [OSDI '16], GraphGrind [ICS '17], Grazelle [PPoPP '18]

Large Graphs

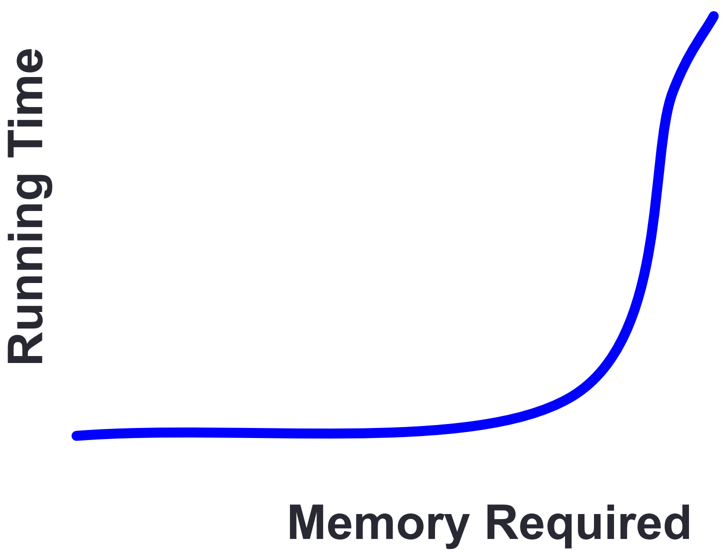
Amazon EC2

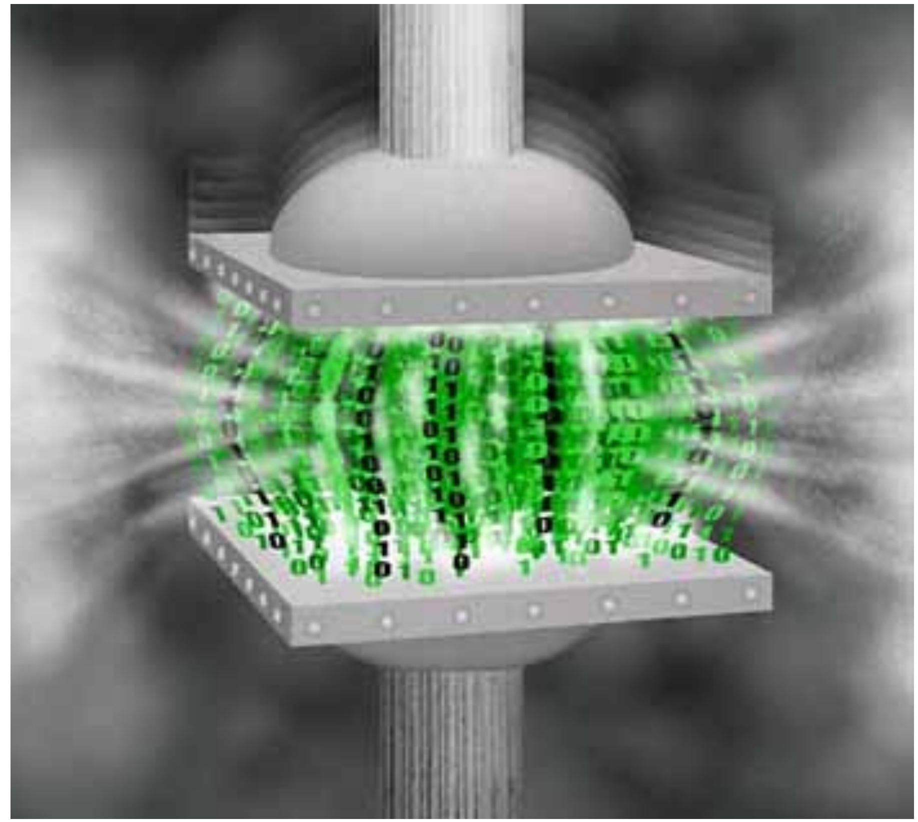
	vCPU	ECU	Memory (GiB)	Instance Storage (GB)	Linux/UNIX Usage
x1e.xlarge	4	12	122	1 x 120 SSD	$0.834 per Hour
x1e.2xlarge	8	23	244	1 x 240 SSD	$1.668 per Hour
x1e.4xlarge	16	47	488	1 x 480 SSD	$3.336 per Hour
x1e.8xlarge	32	91	976	1 x 960	$6.672 per Hour
x1e.16xlarge	64	179	1952	1 x 1920 SSD	$13.344 per Hour
x1e.32xlarge	128	340	3904	2 x 1920 SSD	$26.688 per Hour

- Most can fit on commodity shared memory machine

Example
Dell PowerEdge R930:
Up to 96 cores and 6 TB of RAM

What if you don't have or can't afford that much memory?





Graph Compression

Ligra+: Adding Graph Compression to Ligra

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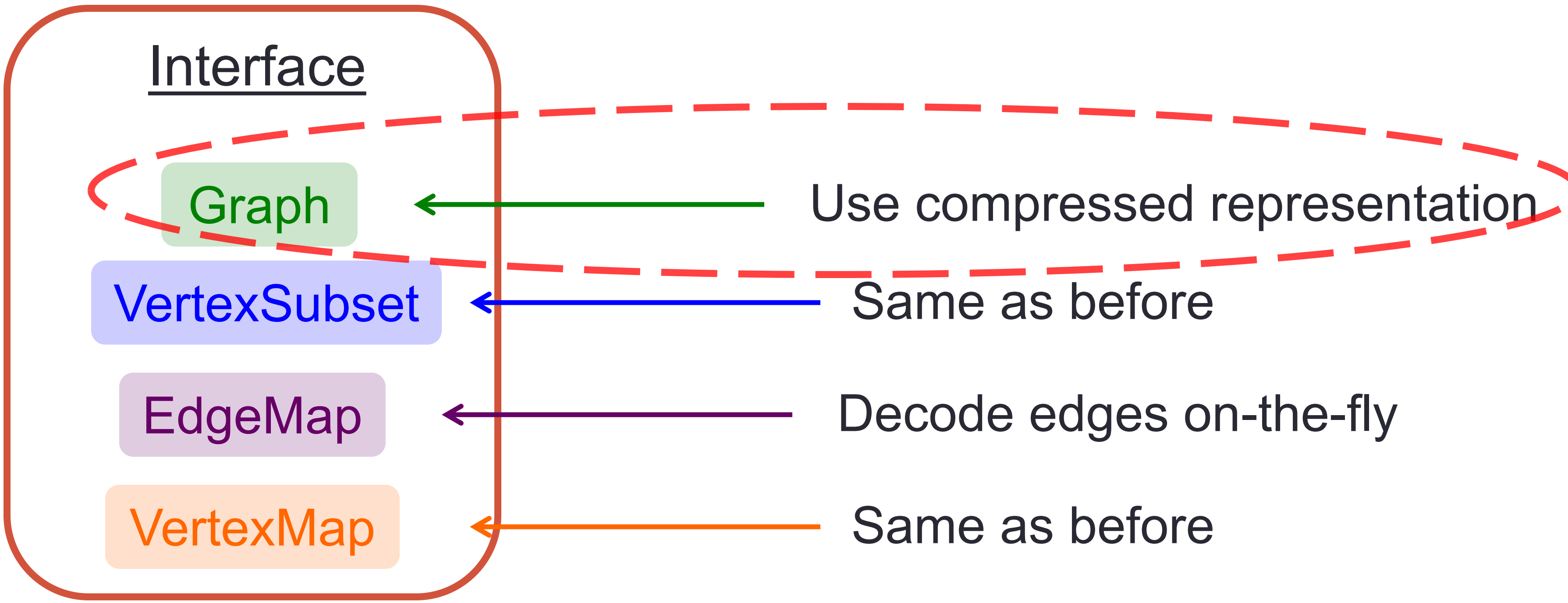
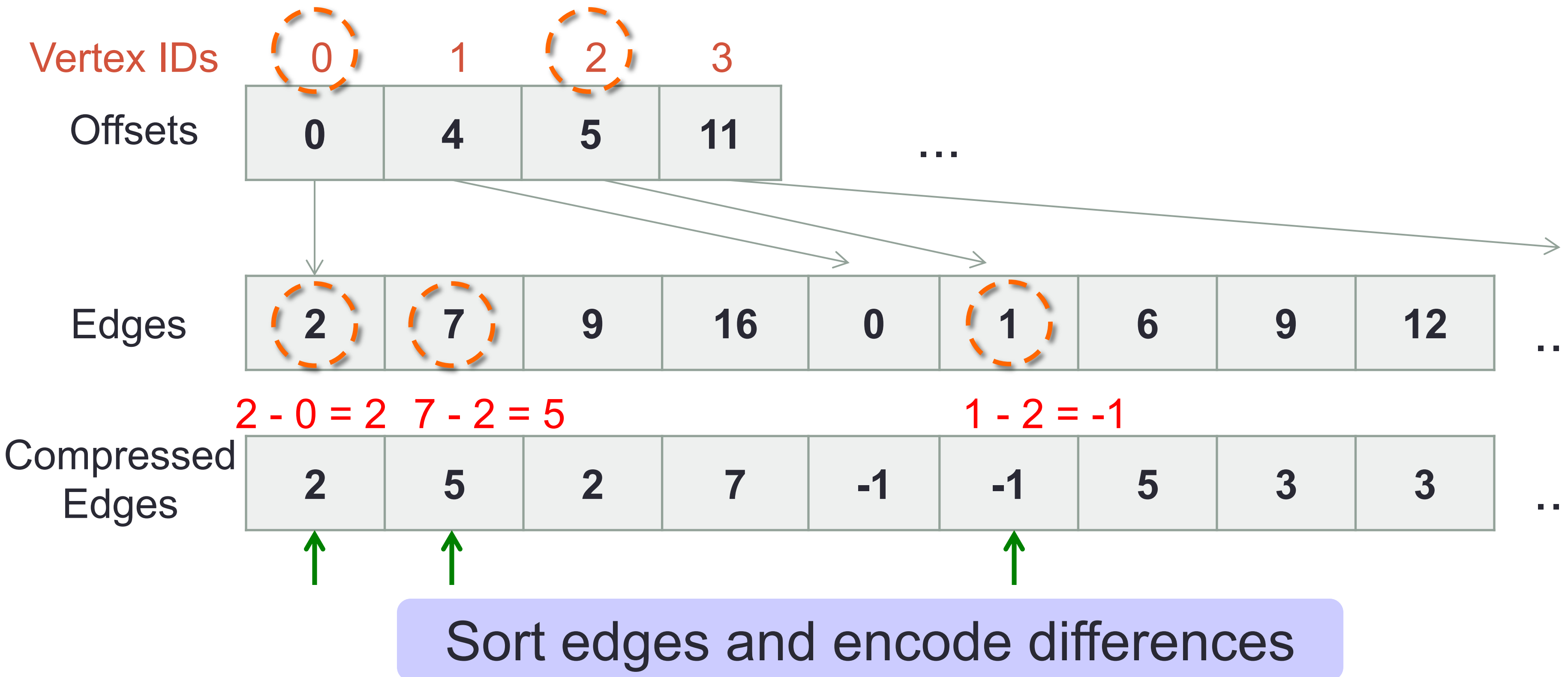


- Same interface as Ligra
- All changes hidden from the user!

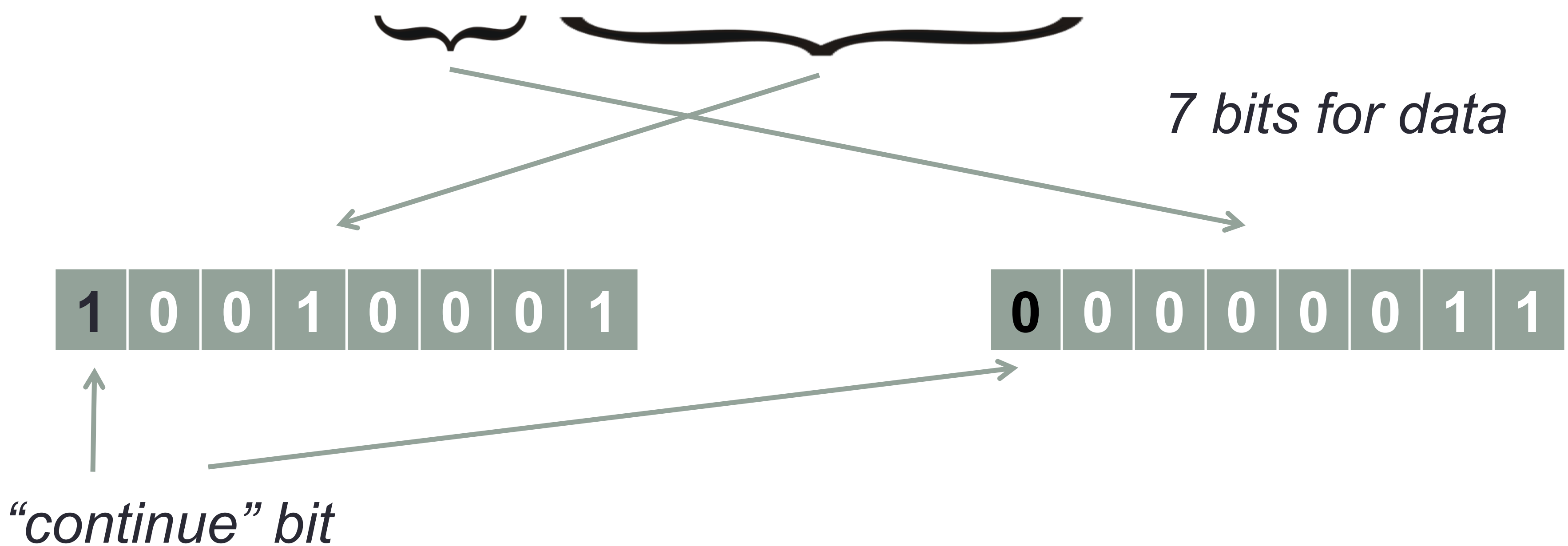
Graph representation



- Graph reordering to improve locality
  - Goal: give neighbors IDs close to vertex ID
  - BFS, DFS, METIS, our own separator-based algorithm

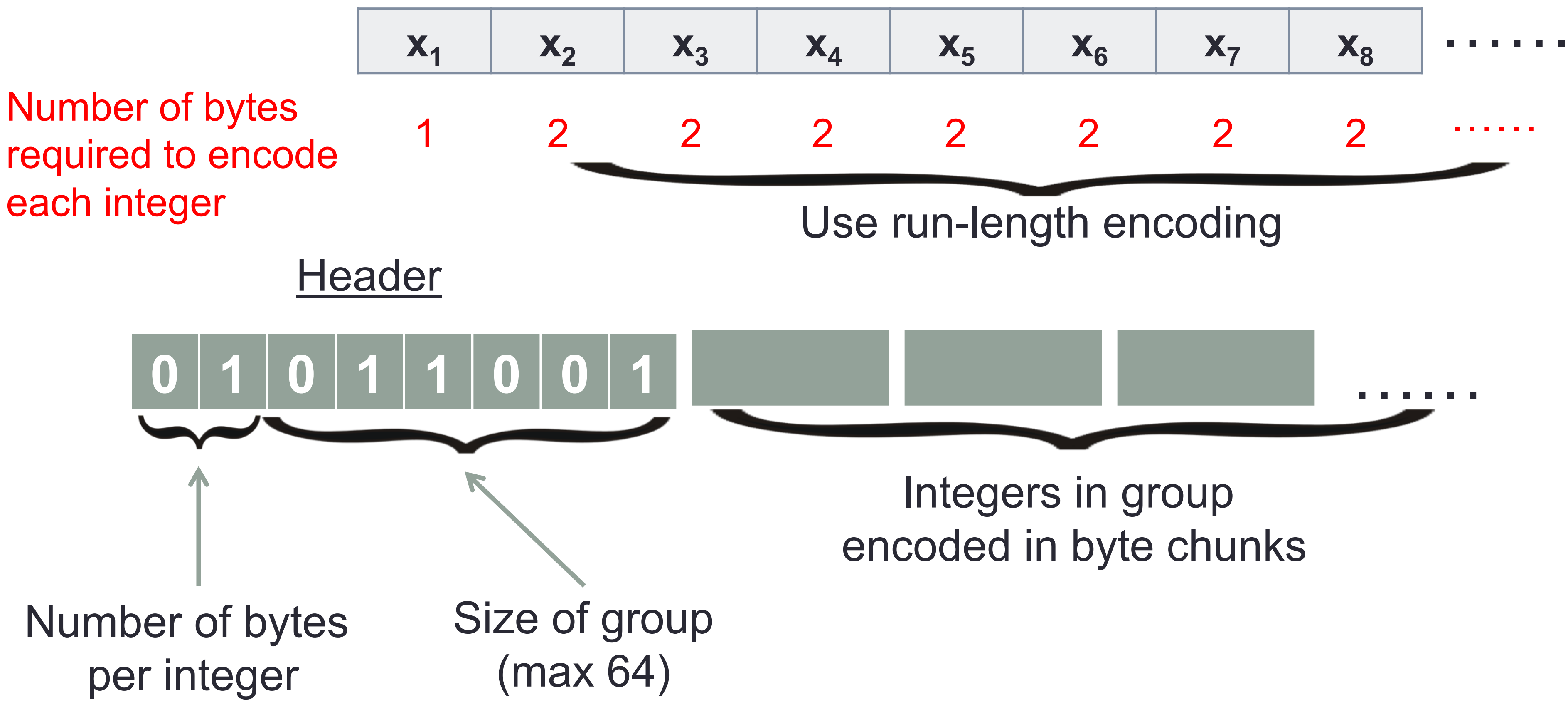
Variable-length codes

- k-bit codes
  - Encode value in chunks of k bits
  - Use k-1 bits for data, and 1 bit as the "continue" bit
- Example: encode "401" using 8-bit (byte) code
- In binary: 1 1 0 0 1 0 0 0 1



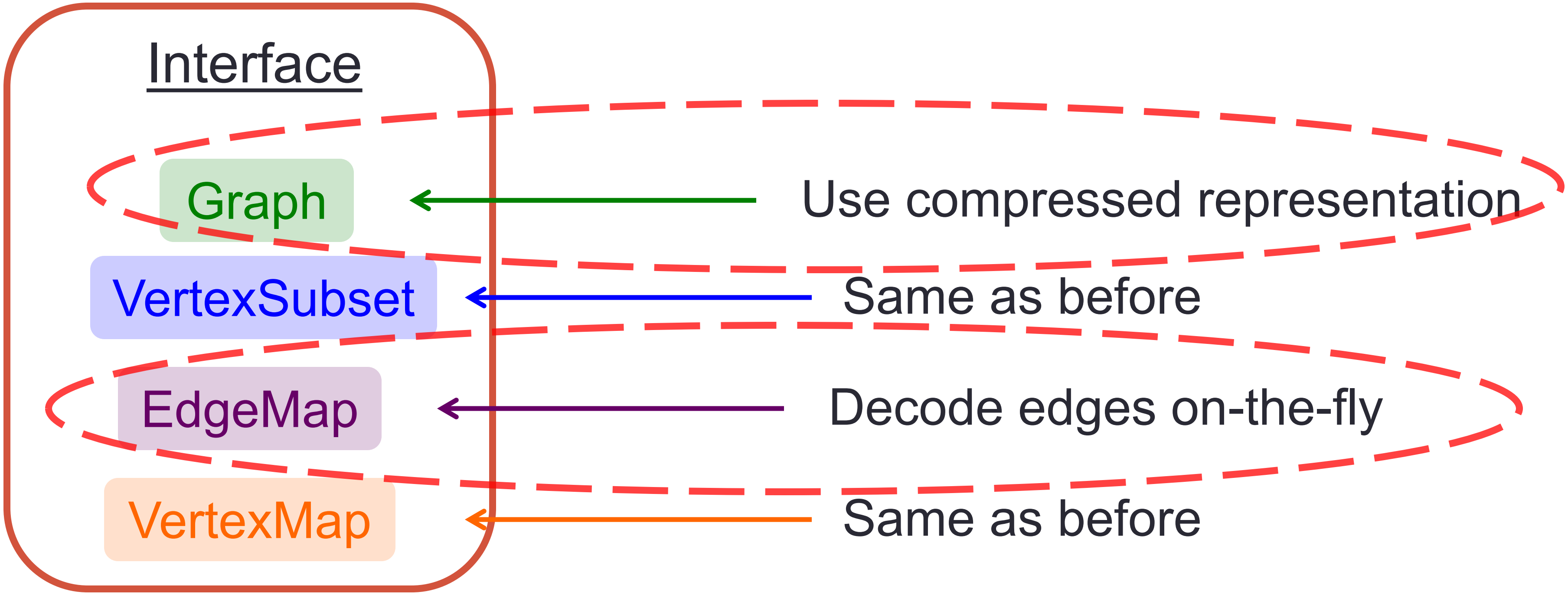
Encoding optimization

- Another idea: get rid of “continue” bits



- Increases space, but makes decoding cheaper (no branch misprediction from checking “continue” bit)

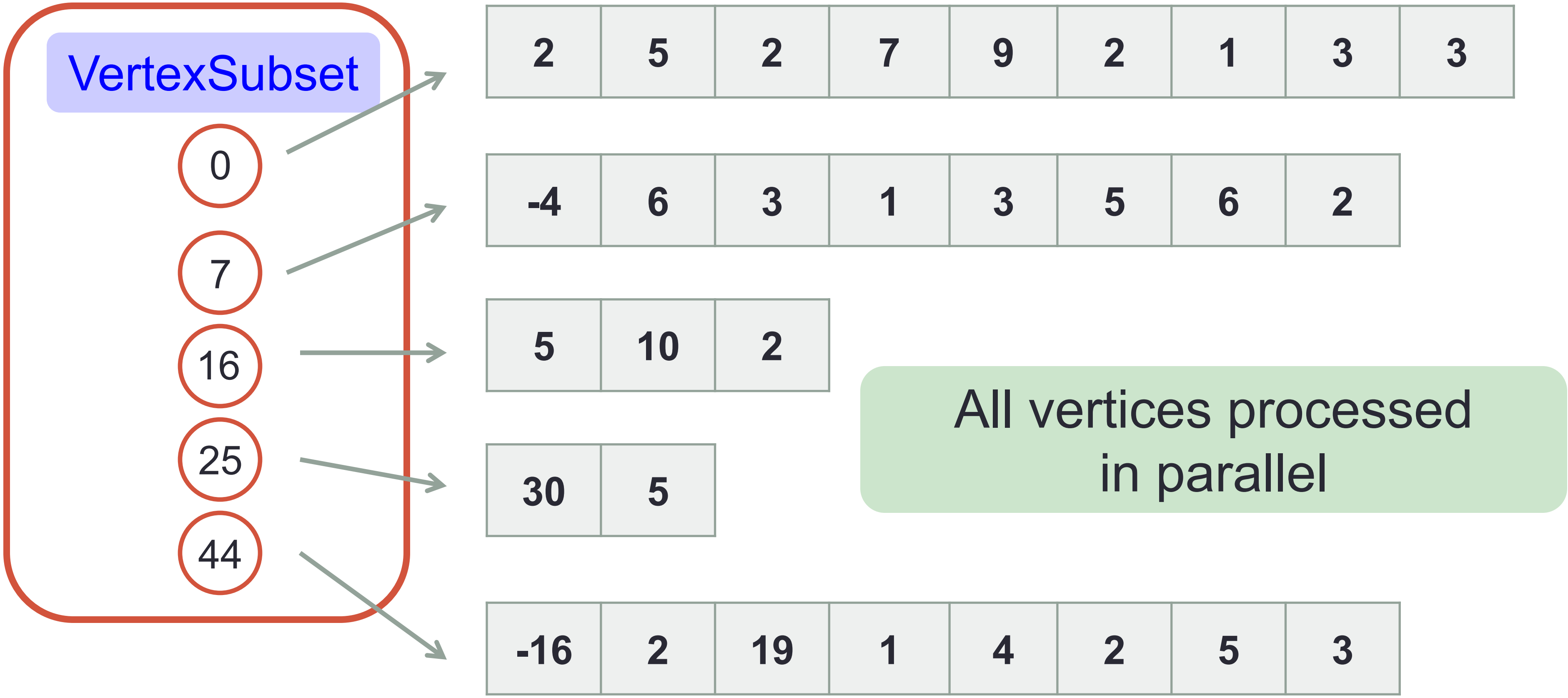
Ligra+: Adding Graph Compression to Ligra



- Same interface as Ligra
- All changes hidden from the user!

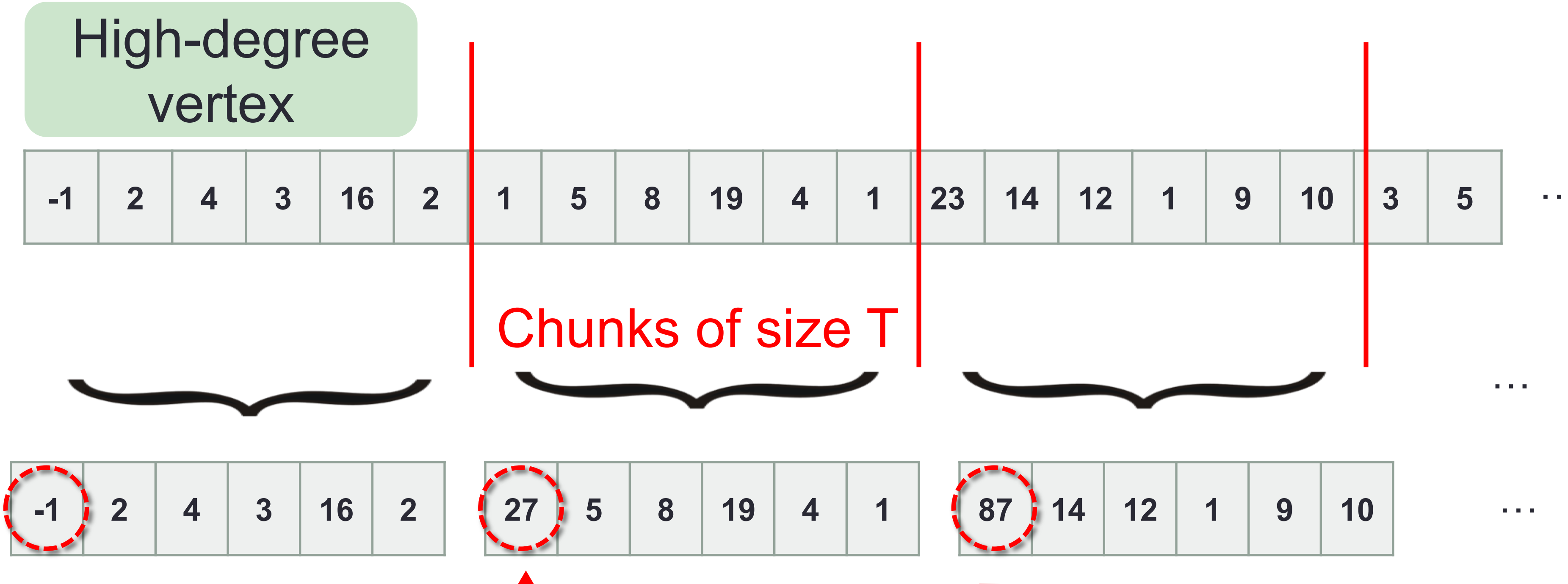
Modifying EdgeMap

- Processes outgoing edges of a subset of vertices



What about high-degree vertices?

Handling high-degree vertices

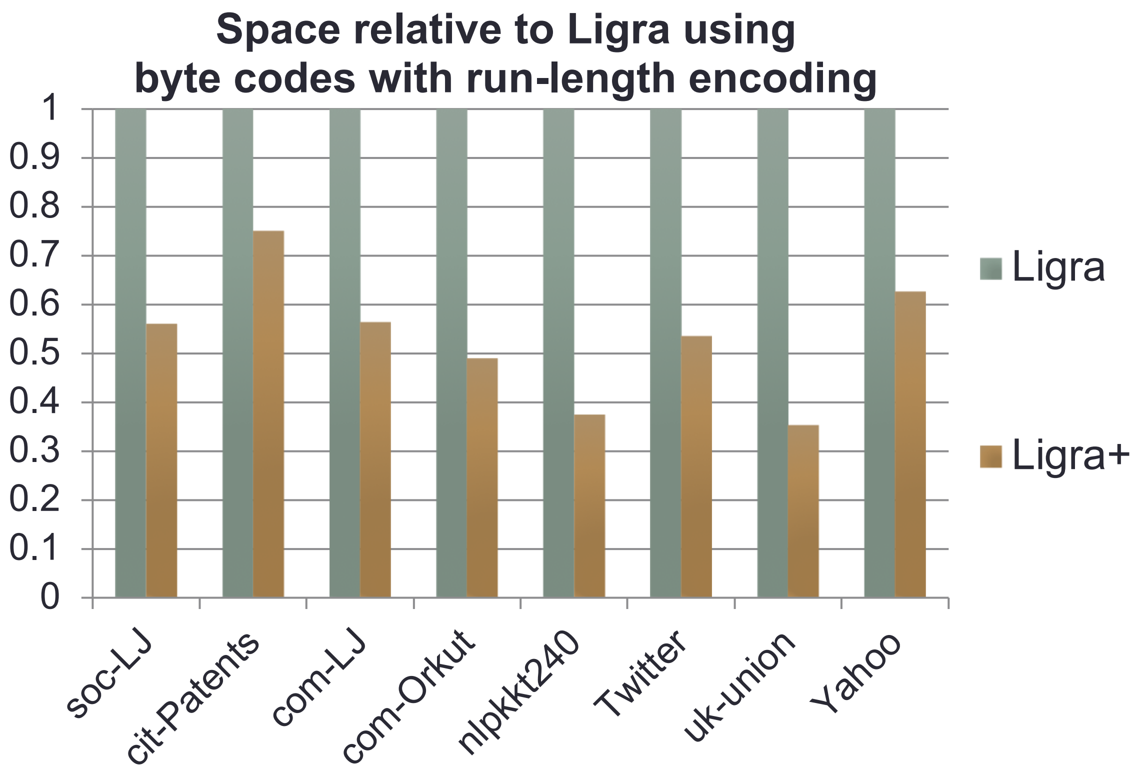


Encode first entry relative to source vertex

All chunks can be decoded in parallel!

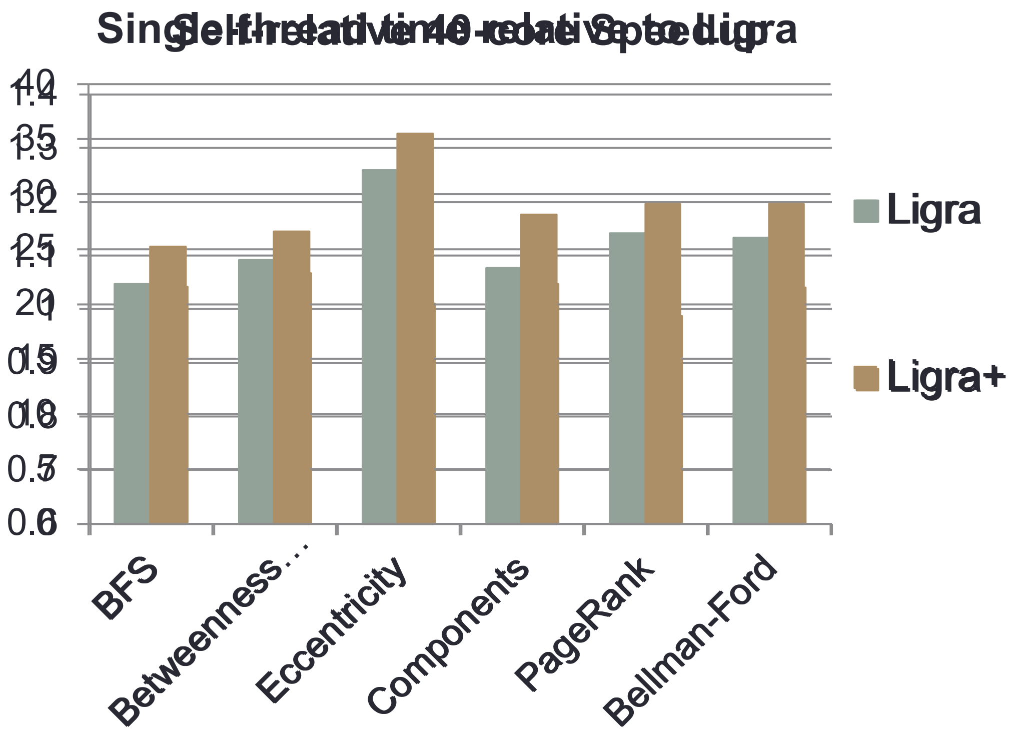
- We chose T=1000
- Similar performance and space usage for a wide range of T

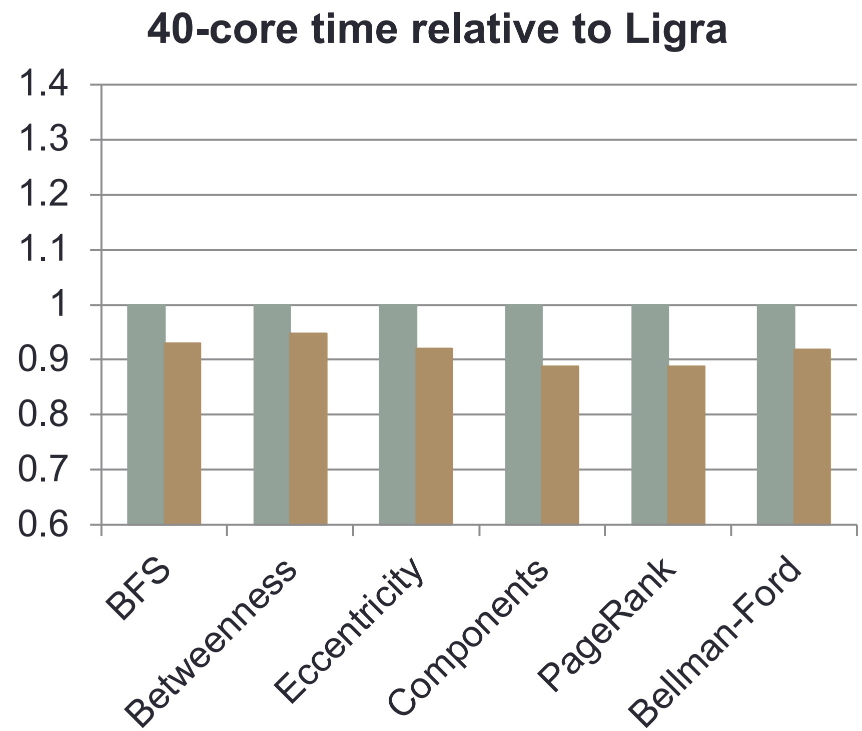
Ligra+ Space Savings



- Space savings of about 1.3—3x
- Could use more sophisticated schemes to further reduce space, but more expensive to decode
- Cost of decoding on-the-fly?

Ligra+ Performance





- Cost of decoding on-the-fly?
- Memory subsystem is a scalability bottleneck in parallel as these graph algorithms are memory-bound
- Ligra+ decoding gets better parallel speed up

Ligra Summary

VertexSubset VertexMap EdgeMap

Optimizations: Hybrid traversal and graph compression

Breadth-first search	Single-source shortest paths
Betweenness centrality	Eccentricity estimation
Connected components	(Personalized) PageRank
Triangle counting	Local graph clustering
K-core decomposition	Biconnected components
Maximal independent set	Collaborative filtering
…	…

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