Exact and Parallel Triangle Counting in Dynamic Graphs

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Outline

Background

- Triangle Counting
- Dynamic Graphs

Implementation

- Graph updating
- Triangle updating

Evaluation

- Batch size
- Breakdown
- Speedup

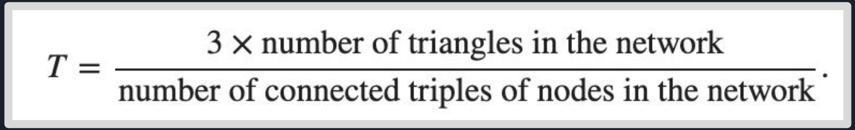
Discussion or Questions

Triangle Counting

Background

Triangle Counting

Applications



$$T = \frac{3 \times \text{number of triangles in the network}}{\text{number of connected triples of nodes in the network}}.$$

- Finding transitivity
- Spam detection in email networks
- Finding tightly knit communities
- Finding trusses k-trusses
- Evaluating the quality of different community detection algorithms

Background
Triangle Counting

Current Approaches

- Enumerating over all node triplets $O(V^3)$
- Using linear algebra operations
- Adjacency list intersection (using hash tables)

Dynamic Graphs

Background

Dynamic Graphs

Useful for larger graphs with evolving datasets

Needs two things

1) Dynamic data structure
2) Algorithm to update the metric of interest

Should be computationally inexpensive compared to restarting the computation from scratch

Should produce the same result as the static graph algorithm

Background

Dynamic Graphs

Existing dynamic graph frameworks

STINGER (DISTINGER for distributed systems and cuSTINGER for GPUs)

AIMS

GraphIN

- Why Stinger?

More flexible than CSR

Supports update operations

Better locality than a linked list

Lower storage bound

Dynamic Graph Updating

Dynamic Graph Updating

Bunch multiple changes to a graph into ‘batches’

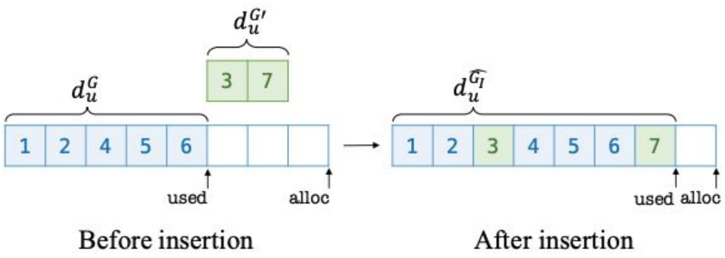
Given a batch update, create an **update-graph** (G’)

Represent the update-graph as a CSR and sort that update-graph

Assuming the original graph was and still is already sorted, merge G’ and G

Dynamic Graph Updating

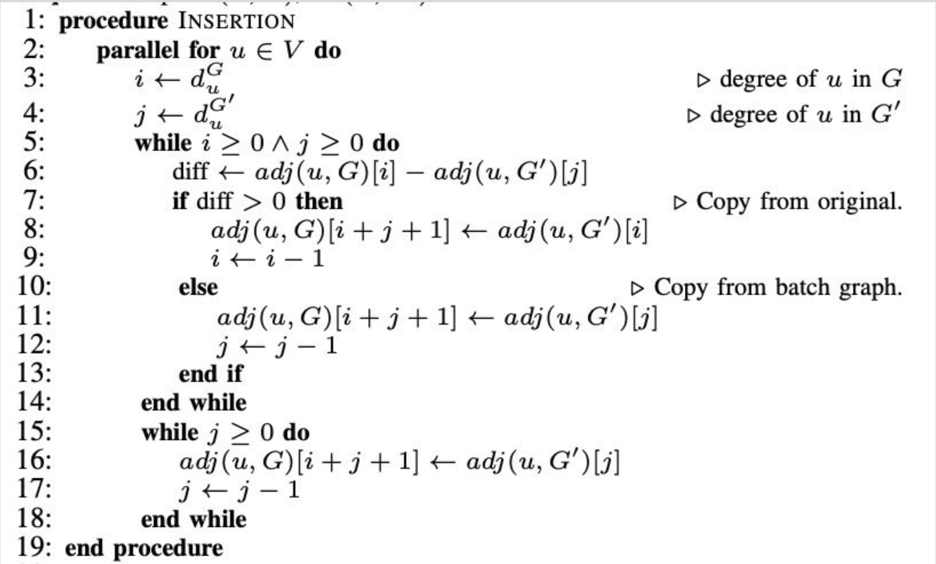
Insertion



Before insertion

After insertion

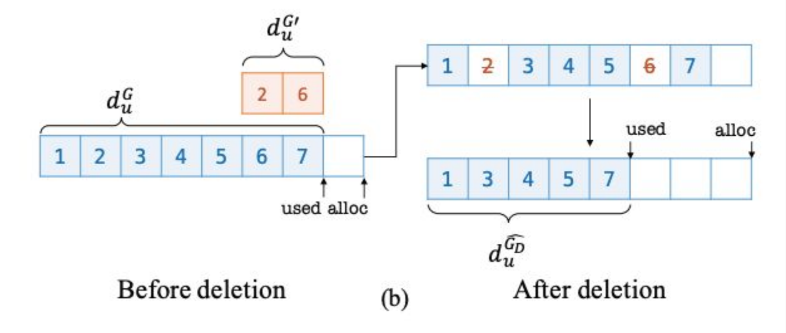
(a)

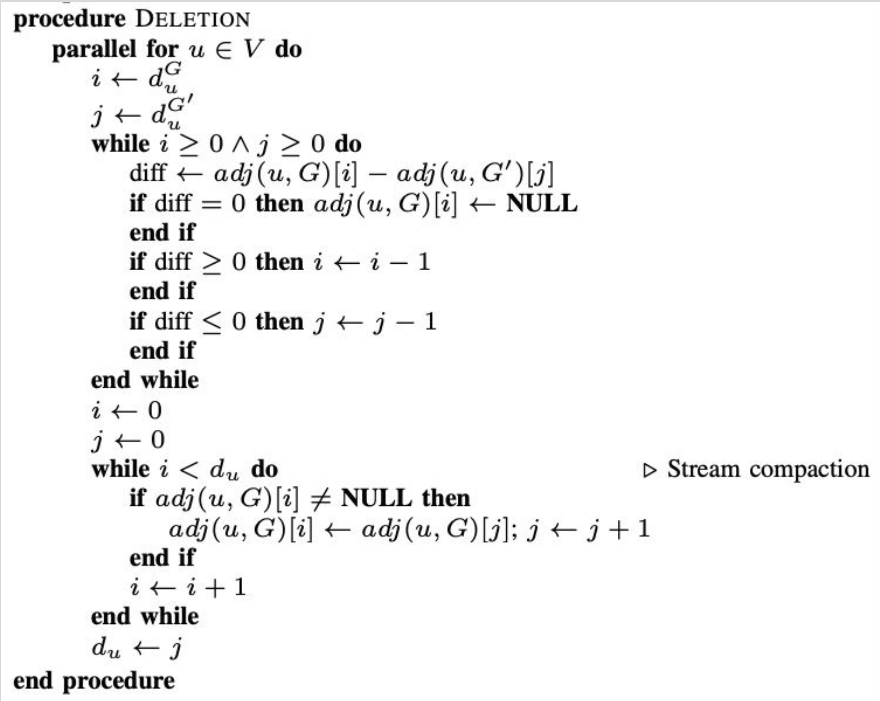


```
procedure INSERTION
    parallel for $u \in V$ do
        $i \leftarrow d_u^G$                                          ▷ degree of $u$ in $G$
        $j \leftarrow d_u^{G'}$                                        ▷ degree of $u$ in $G'$
        while $i \geq 0 \land j \geq 0$ do
            diff $\leftarrow adj(u, G)[i] - adj(u, G')[j]$
            if diff $> 0$ then                                   ▷ Copy from original.
                $adj(u, G)[i + j + 1] \leftarrow adj(u, G')[i]$
                $i \leftarrow i - 1$
            else                                                ▷ Copy from batch graph.
                $adj(u, G)[i + j + 1] \leftarrow adj(u, G')[j]$
                $j \leftarrow j - 1$
            end if
        end while
        while $j \geq 0$ do
            $adj(u, G)[i + j + 1] \leftarrow adj(u, G')[j]$
            $j \leftarrow j - 1$
        end while
end procedure
```

Dynamic Graph Updating

Deletion





```
procedure DELETION
    parallel for $u \in V$ do
        $i \leftarrow d_u^G$
        $j \leftarrow d_u^{G'}$
        while $i \geq 0 \wedge j \geq 0$ do
            diff $\leftarrow adj(u, G)[i] - adj(u, G')[j]$
            if diff $= 0$ then $adj(u, G)[i] \leftarrow$ NULL
            end if
            if diff $\geq 0$ then $i \leftarrow i - 1$
            end if
            if diff $\leq 0$ then $j \leftarrow j - 1$
            end if
        end while
        $i \leftarrow 0$
        $j \leftarrow 0$
        while $i < d_u$ do                                        ▷ Stream compaction
            if $adj(u, G)[i] \neq$ NULL then
                $adj(u, G)[i] \leftarrow adj(u, G)[j]$; $j \leftarrow j + 1$
            end if
            $i \leftarrow i + 1$
        end while
        $d_u \leftarrow j$
end procedure
```

Triangle Count Updating

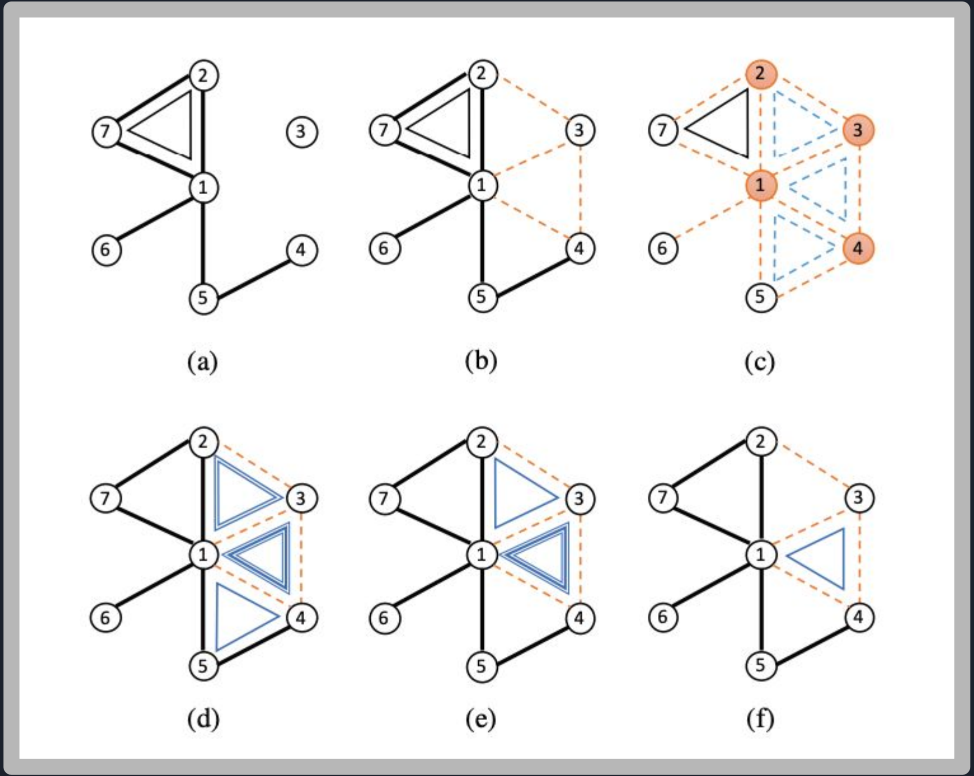
Triangle Counting

Types of Triangles

- Δ^i_1 (triangles with 1 new edge and 2 old edges)
- Δ^i_2 (triangles with 2 new edges and 1 old edge)
- Δ^i_3 (triangles with 3 new edges)

$$\boxed{\text{NewTriangles} = |\Delta^i_1| + |\Delta^i_2| + |\Delta^i_3|}$$

Triangle Counting



(a) (b) (c)

(d) (e) (f)

Break up discovery by num new edges

For each edge <u,v> in the batch update, intersect the adjacency lists

$$s_{e,1} = adj(u, \widehat{G_I}) \cap adj(v, \widehat{G_I})$$

$$S_1^i = 2 \cdot |\Delta_1^i| + 4 \cdot |\Delta_2^i| + 6 \cdot |\Delta_3^i|$$

$$S_2^i = \sum_{e \in E'} |s_{e,2}| = 2 \cdot |\Delta_2^i| + 6 \cdot |\Delta_3^i|$$

$$S_3^i = 6 \cdot |\Delta_3^i|$$

Triangle Counting

After gathering and discover 1+, 2+ and 3+ new-edged triangles

Use Inclusion - Exclusion formula to compute total new triangles

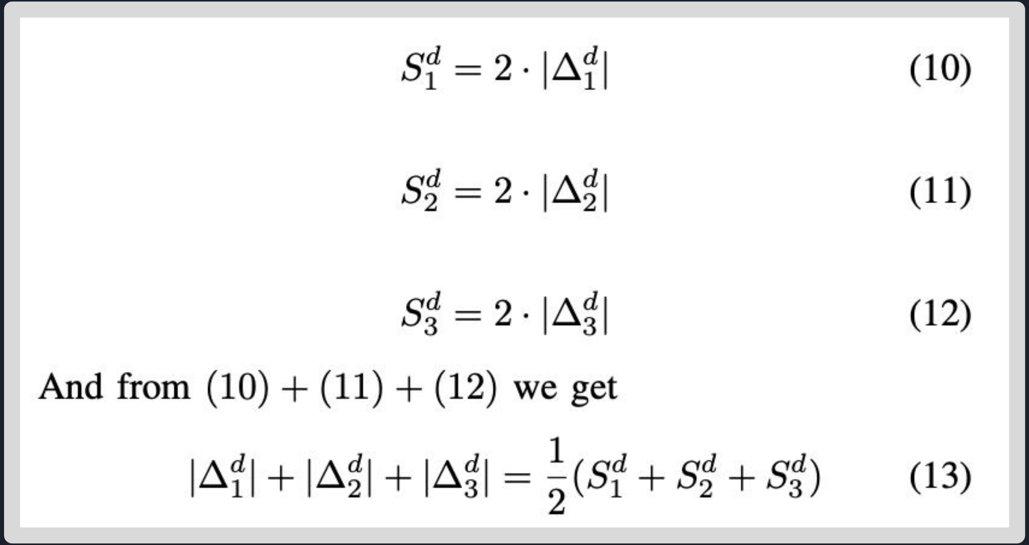
$$|\Delta_1^i| + |\Delta_2^i| + |\Delta_3^i| = \frac{1}{2}\left(S_1^i - S_2^i + \frac{S_3^i}{3}\right)$$

Triangle Counting

Deleting edges is simpler

Look for edges that existed before their removal

Triangles do not get recounted



$$S_1^d = 2 \cdot |\Delta_1^d| \tag{10}$$

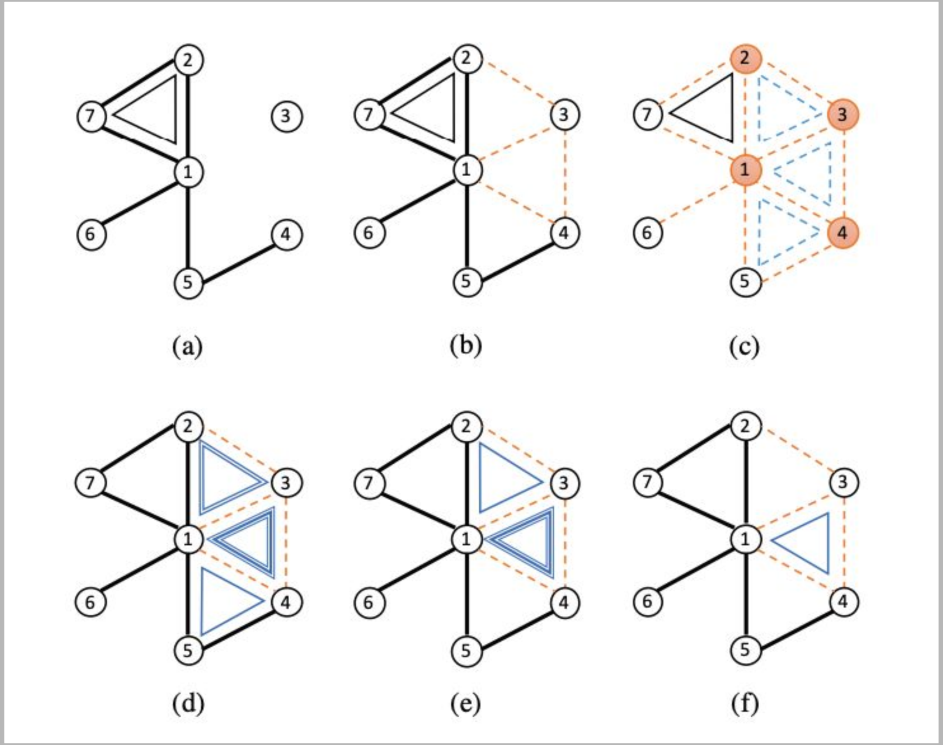
$$S_2^d = 2 \cdot |\Delta_2^d| \tag{11}$$

$$S_3^d = 2 \cdot |\Delta_3^d| \tag{12}$$

And from $(10) + (11) + (12)$ we get

$$|\Delta_1^d| + |\Delta_2^d| + |\Delta_3^d| = \frac{1}{2}(S_1^d + S_2^d + S_3^d) \tag{13}$$

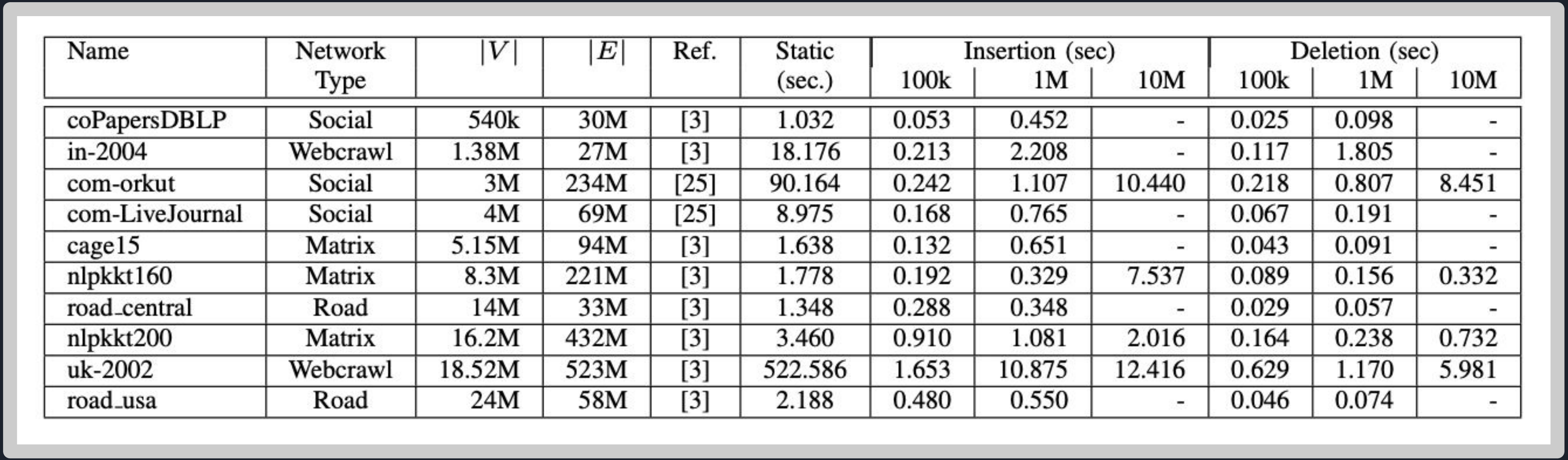
Triangle Counting



Evaluation

Evaluation

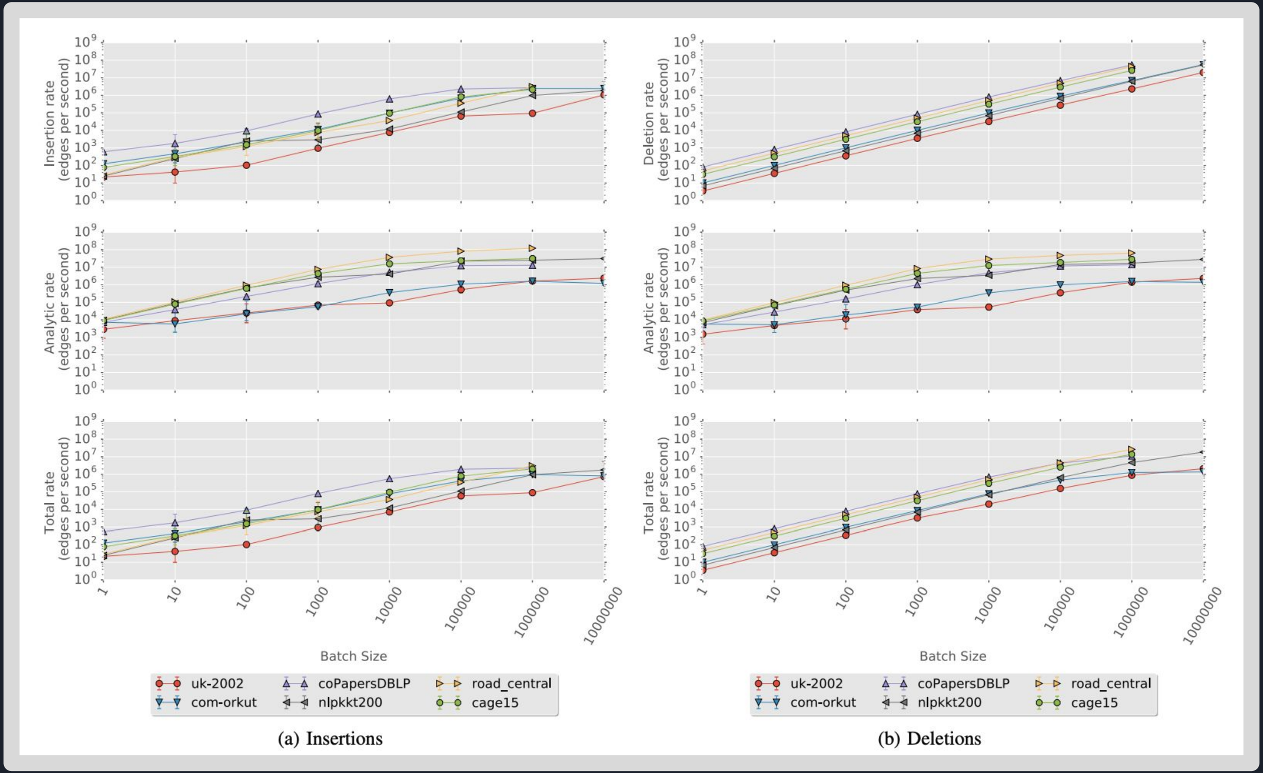
Networks Used



Name	Network Type	$\lvert V\rvert$	$\lvert E\rvert$	Ref.	Static (sec.)	Insertion (sec)			Deletion (sec)		
						100k	1M	10M	100k	1M	10M
coPapersDBLP	Social	540k	30M	[3]	1.032	0.053	0.452	-	0.025	0.098	-
in-2004	Webcrawl	1.38M	27M	[3]	18.176	0.213	2.208	-	0.117	1.805	-
com-orkut	Social	3M	234M	[25]	90.164	0.242	1.107	10.440	0.218	0.807	8.451
com-LiveJournal	Social	4M	69M	[25]	8.975	0.168	0.765	-	0.067	0.191	-
cage15	Matrix	5.15M	94M	[3]	1.638	0.132	0.651	-	0.043	0.091	-
nlpkkt160	Matrix	8.3M	221M	[3]	1.778	0.192	0.329	7.537	0.089	0.156	0.332
road_central	Road	14M	33M	[3]	1.348	0.288	0.348	-	0.029	0.057	-
nlpkkt200	Matrix	16.2M	432M	[3]	3.460	0.910	1.081	2.016	0.164	0.238	0.732
uk-2002	Webcrawl	18.52M	523M	[3]	522.586	1.653	10.875	12.416	0.629	1.170	5.981
road_usa	Road	24M	58M	[3]	2.188	0.480	0.550	-	0.046	0.074	-

Evaluation

Batch Size

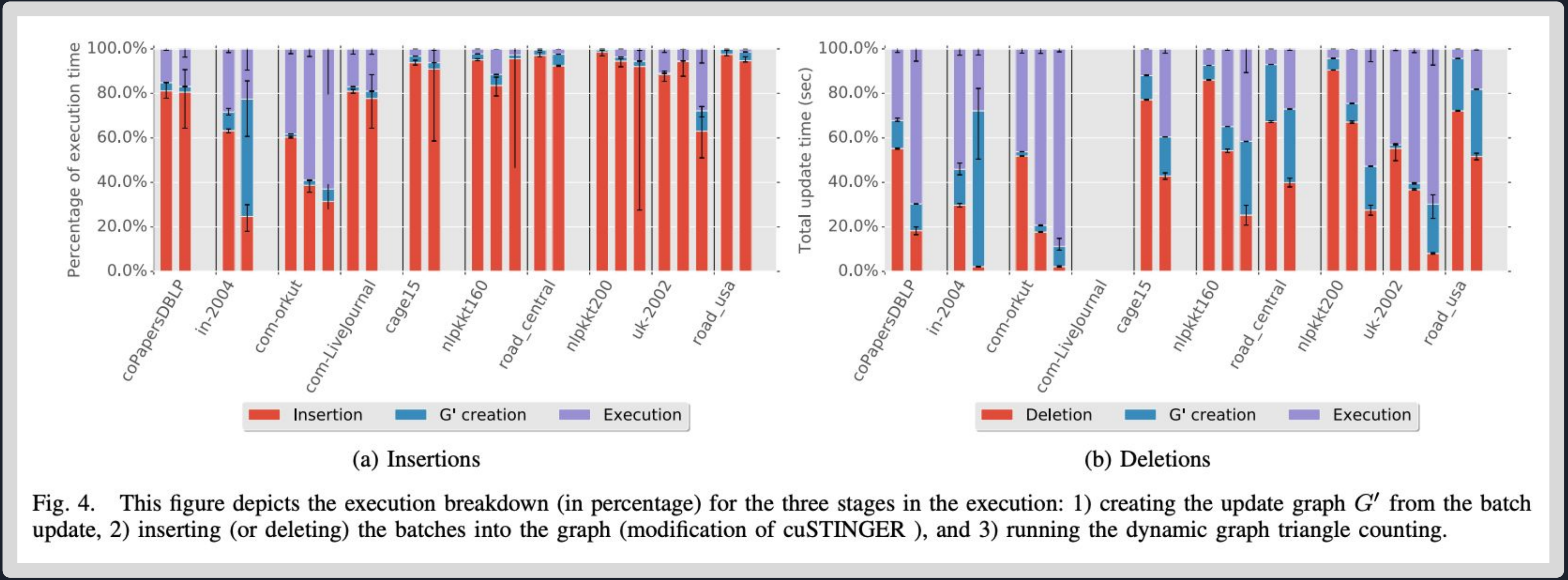


(a) Insertions

(b) Deletions

Evaluation

Breakdown



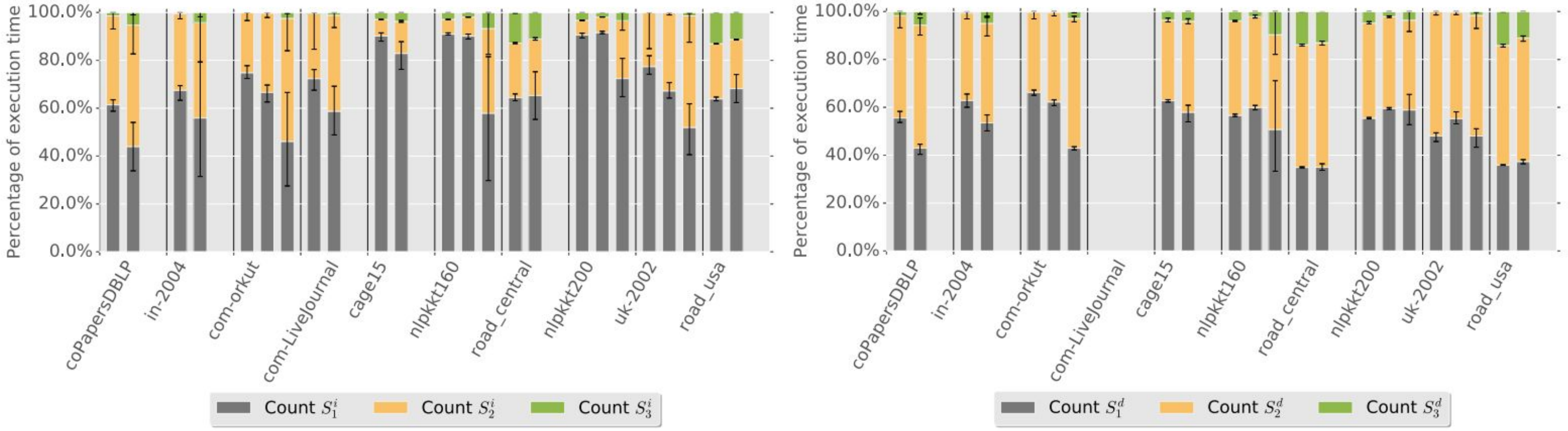
(a) Insertions

(b) Deletions

Fig. 4. This figure depicts the execution breakdown (in percentage) for the three stages in the execution: 1) creating the update graph G' from the batch update, 2) inserting (or deleting) the batches into the graph (modification of cuSTINGER), and 3) running the dynamic graph triangle counting.

Evaluation

Breakdown



(a) Insertions

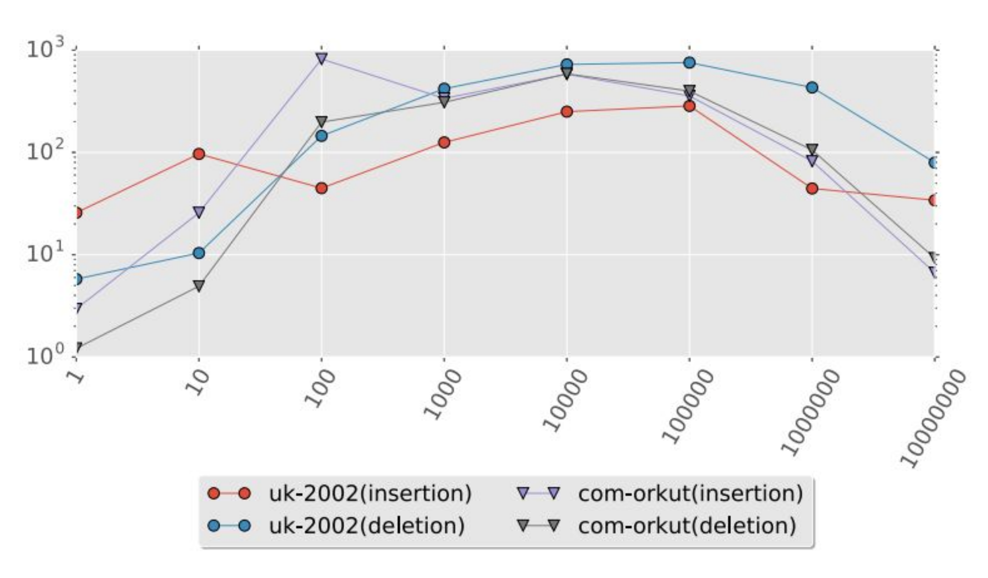
(b) Deletions

Fig. 5. This figure depicts the execution breakdown (in percentage) of only the dynamic triangle counting analytic using the inclusion-exclusion formulation. For both the insertion (a) and deletions (b) there are three phases. The execution time of the triangle counting accounts for the purple bars in Fig. 4.

Evaluation

Speedup

Speedup - compared to previous algorithm which recounted all triangles after each update



Questions?