Analysis of Work-Stealing Scheduler

Yan Gu

6.886 Algorithm Engineering

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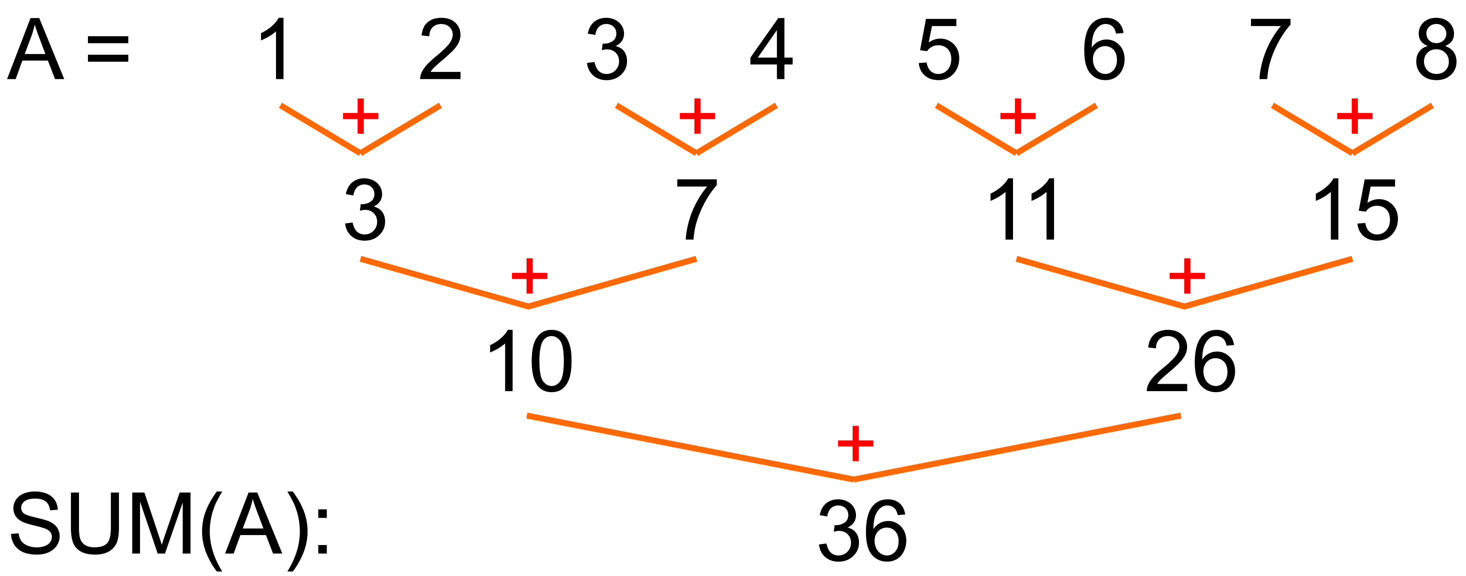
Shared-memory multicore parallelism

64-100+ cores
High-end servers

6-8 cores
Smartphones

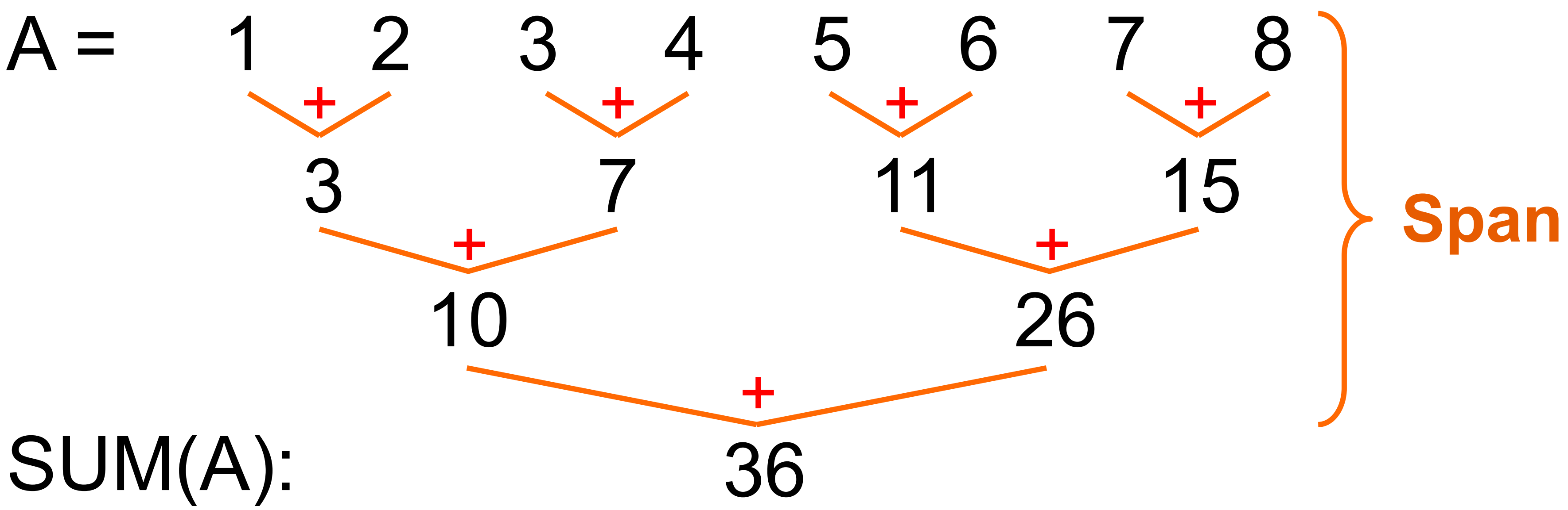
16-40 cores
PC/Workstation

50-100x increase in the last 15 years!




```
Function SUM(A)
    If  |A| = 1  then return  A(1)
    In Parallel
        a = SUM(first half of A)
        b = SUM(second half of A)
    return a + b
```

Cilk
OpenMP
X10
TBB
Habanero
Java fork-join

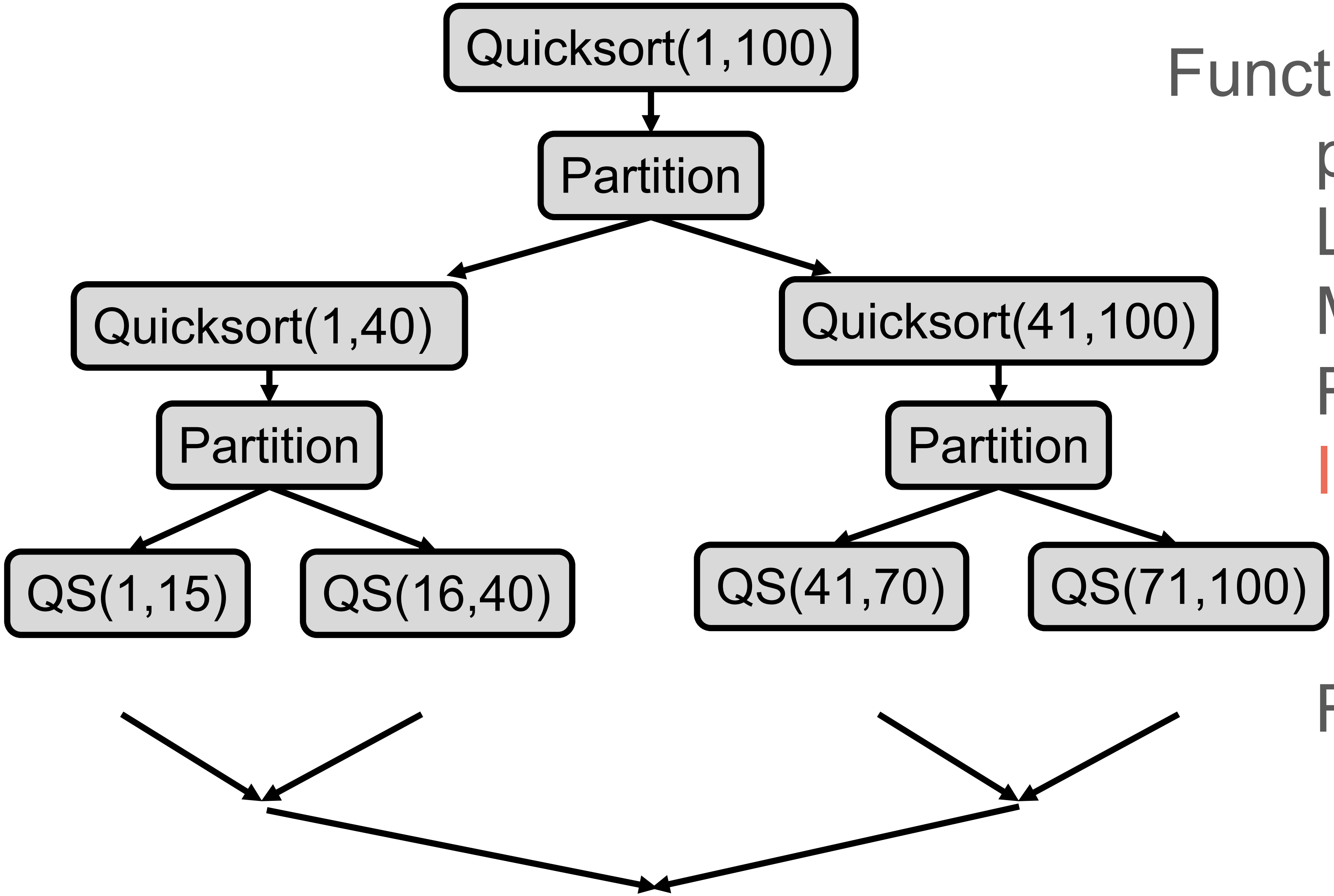


- **Work (W)**: the number of operations (ideally it should match the best sequential solution)

Less overhead

- **Span (D)**: the longest dependence in this computation (ideally to be polylogarithmic)

Better scalability




```
Function QuickSort (A)
    p ← random pivot
    L ← Select (A, <p)
    M ← Select (A, =p)
    R ← Select (A, >p)
    In parallel
        QuickSort (L)
        QuickSort (R)
    Return L + M + R
```

```
parallel_for (int i=0; i<n; i++)
        a[i] = f(a[i]);
```

How is your code actually executed on hardware?

Why analyzing work and span?

```
parallel_for (int i=0; i<n; i++)
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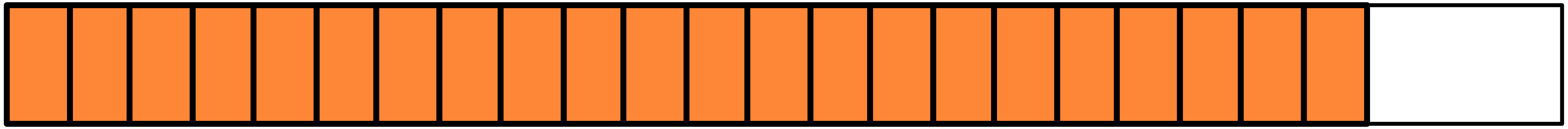
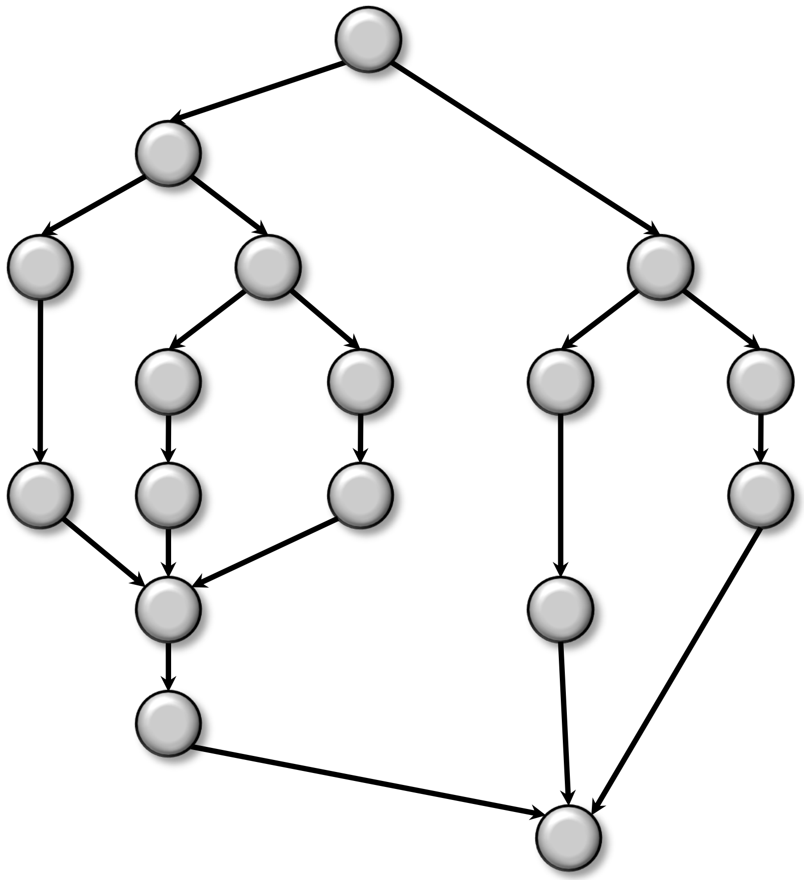


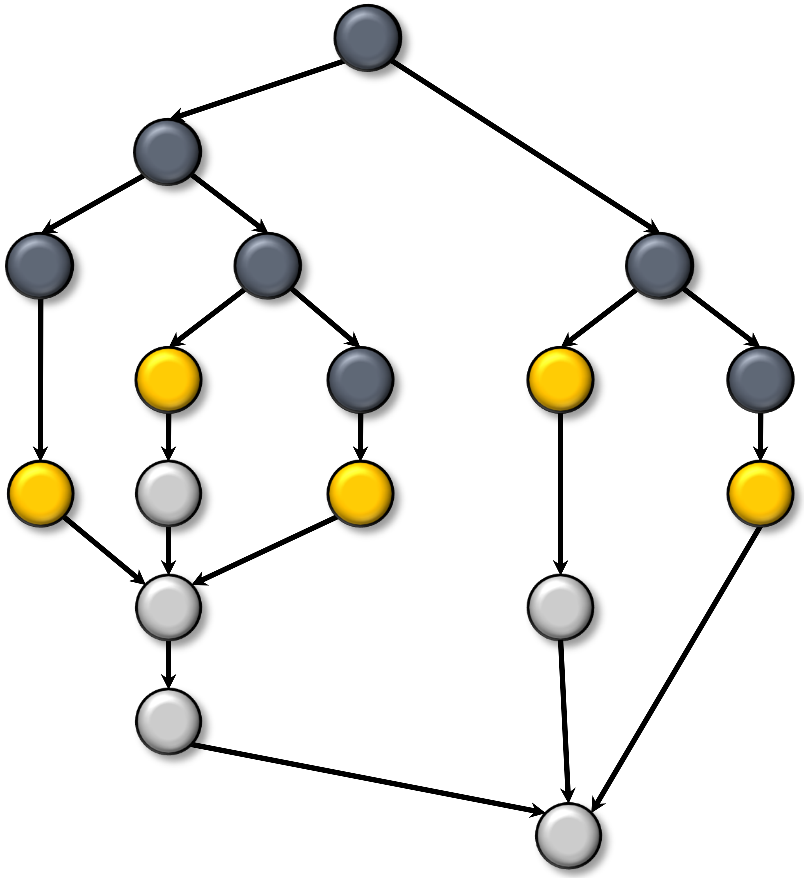
Treat the computation as a DAG

Function SUM(A)
    If |A| = 1 then return A(1)
    In Parallel
        a = SUM(first half of A)
        b = SUM(second half of A)
    return a + b



Greedy scheduler

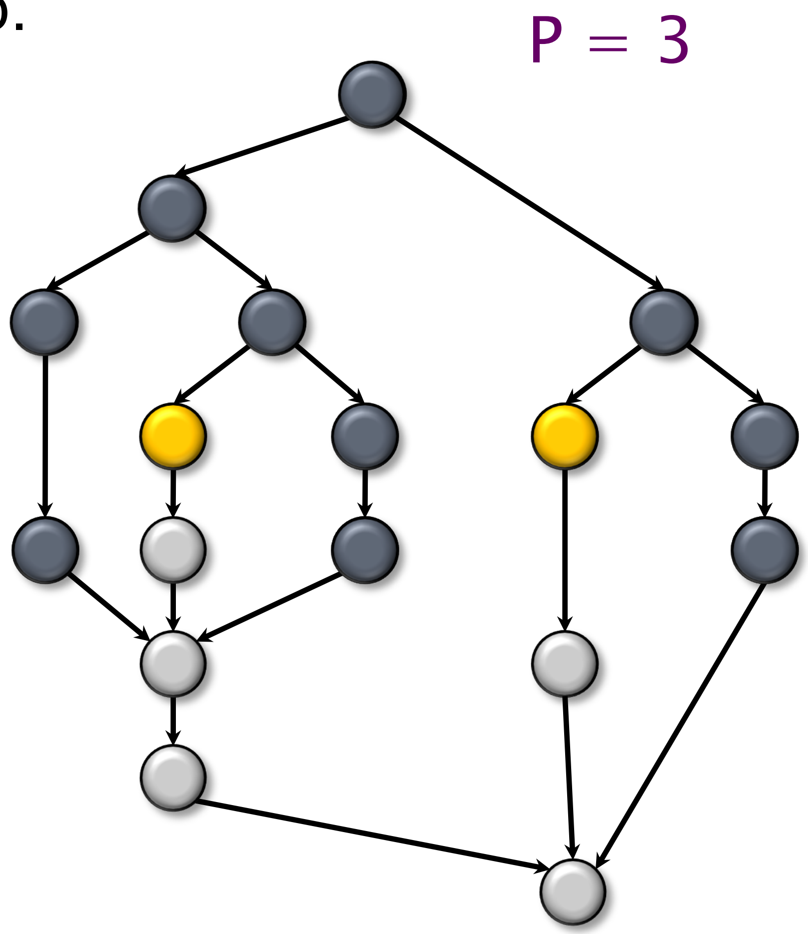
IDEA: Do as much as possible on every step.



Greedy scheduler

IDEA: Do as much as possible on every step.

Either execute p operations

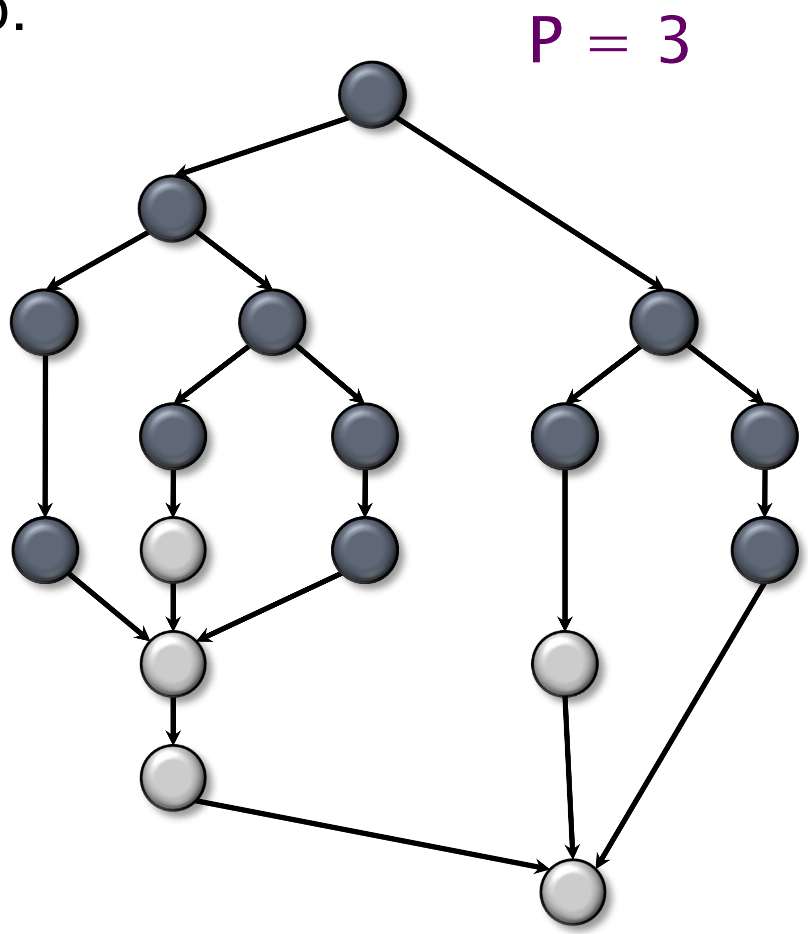


Greedy scheduler

IDEA: Do as much as possible on every step.

Either execute p operations
Or reduce the span by 1

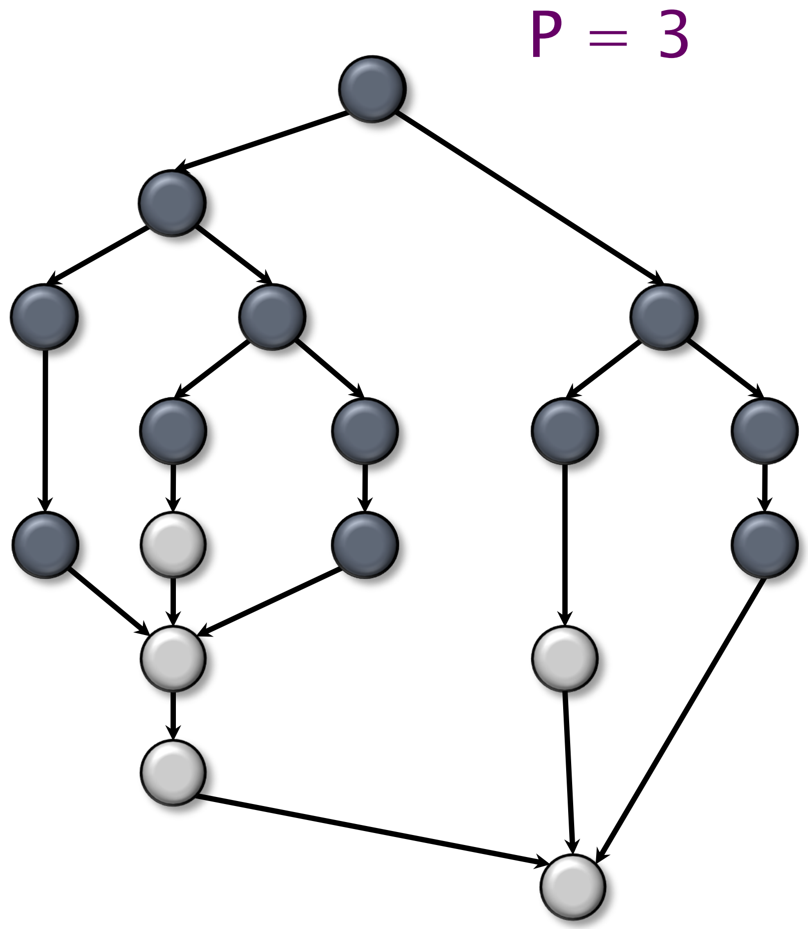
$$T \leq \frac{W}{P} + D$$



Greedy scheduler

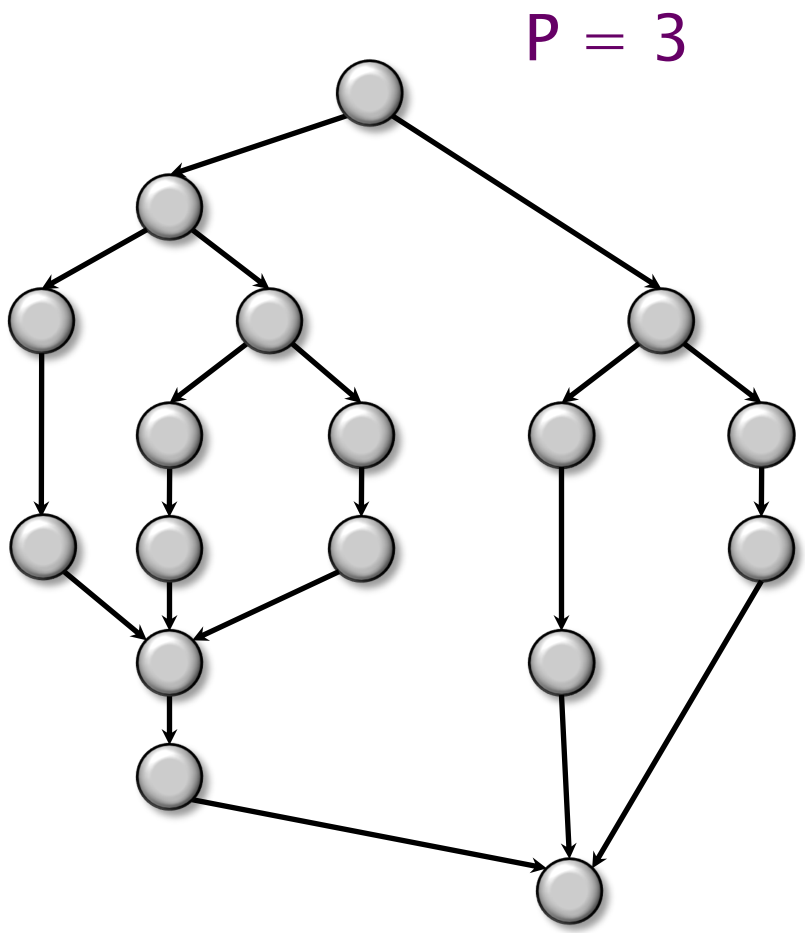
Impractical:

- Assumes processors/threads run in lockstep
- Big overhead in context switching
- Different operations have very different costs

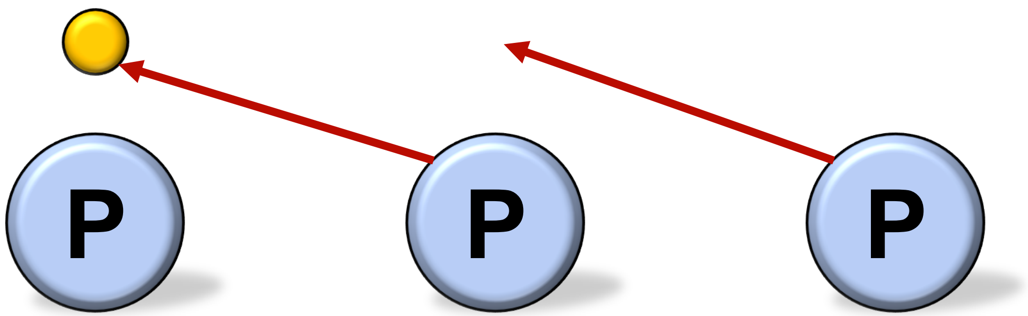


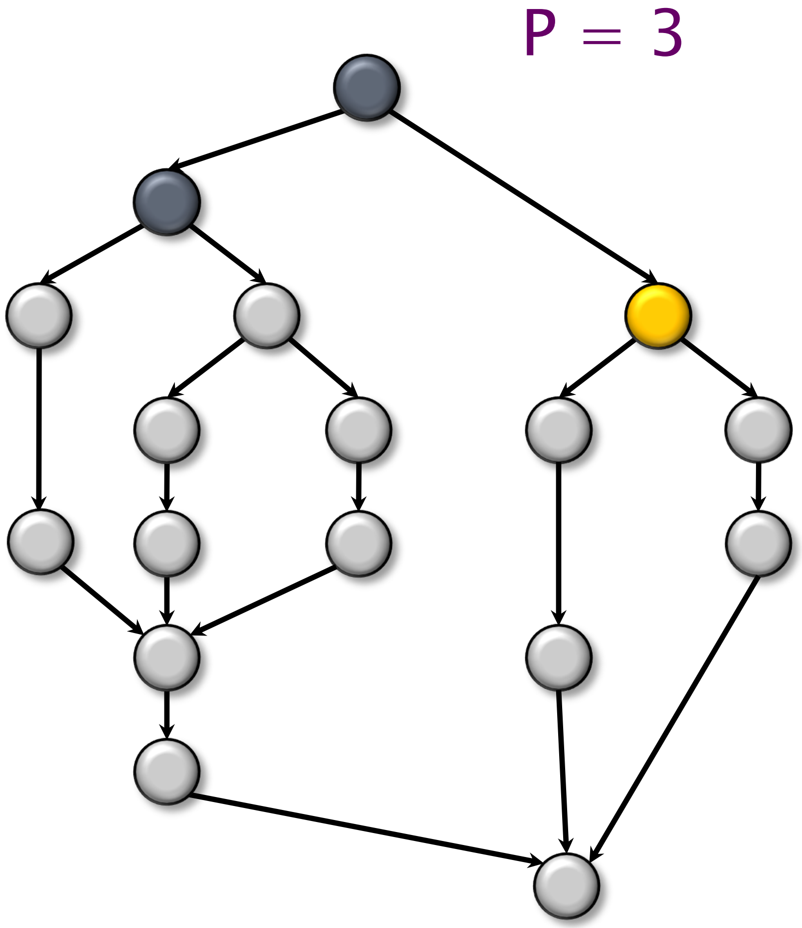
Work-stealing scheduler

- Full details in 6.172: Performance Engineering of Software Systems (Cilk implementation)
- If a processor spawns tasks at a FORK, it continues execution with one of the spawned subtasks, and push the other subtask to the front its queue
- If a processor completes a task, it tries to pull a task from the front of its own queue
- If a processor finishes all tasks in its own queue, it randomly selects another processor, and steals a task from the end of the victim queue (retry if failed)

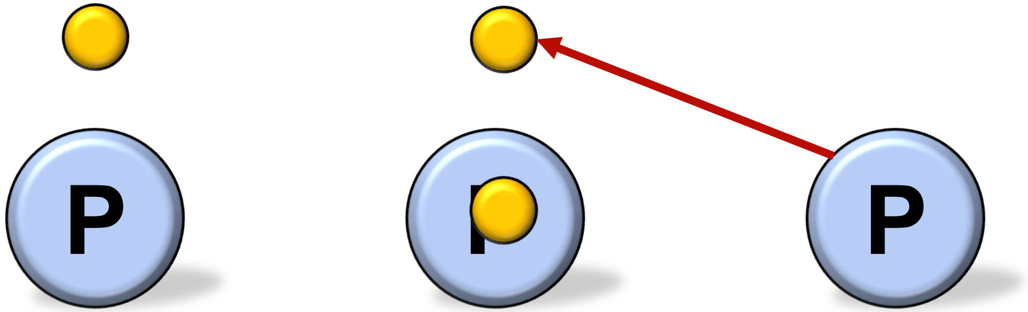


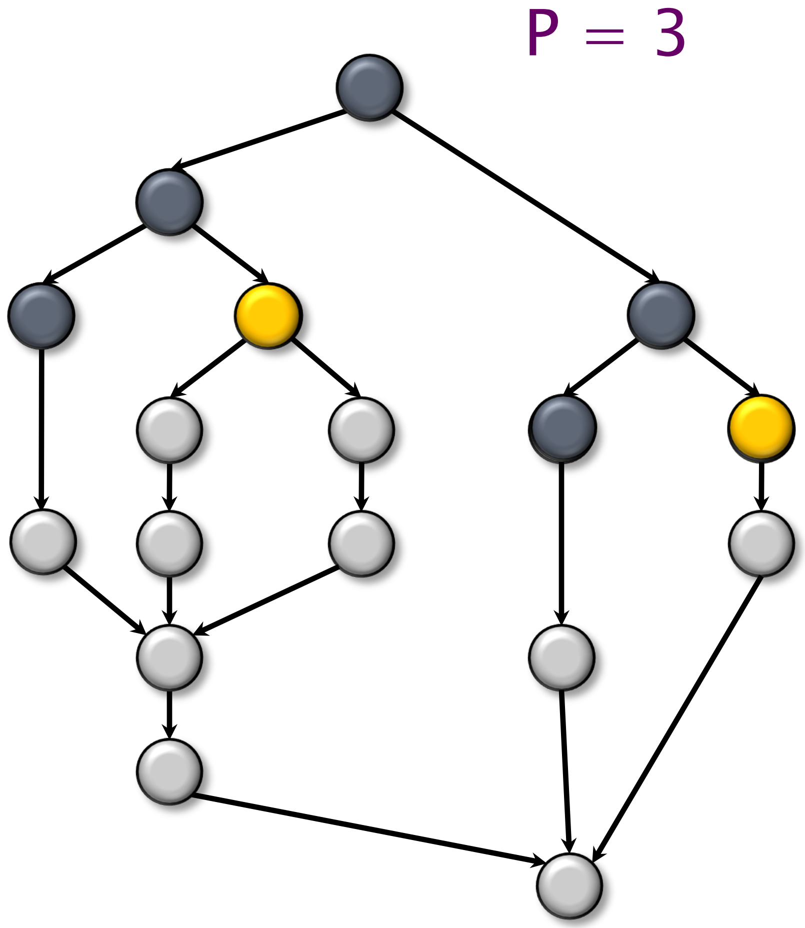
Work-stealing scheduler





Work-stealing scheduler





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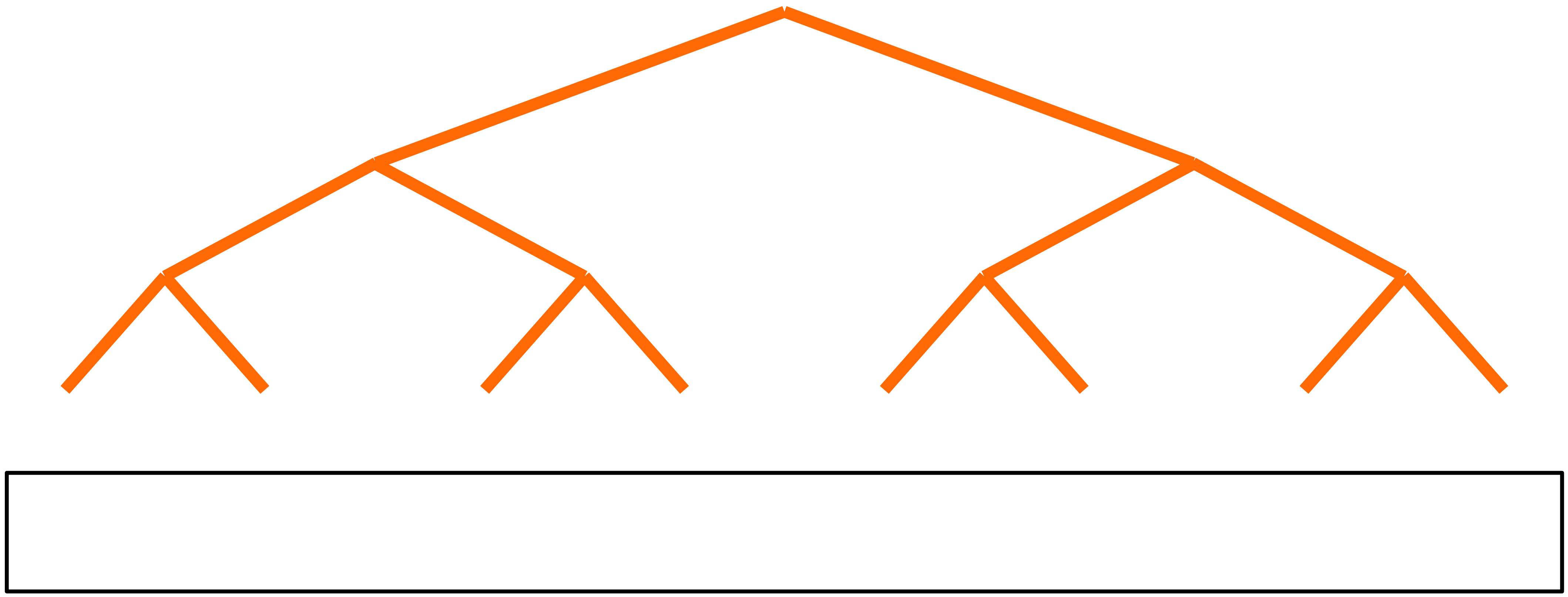
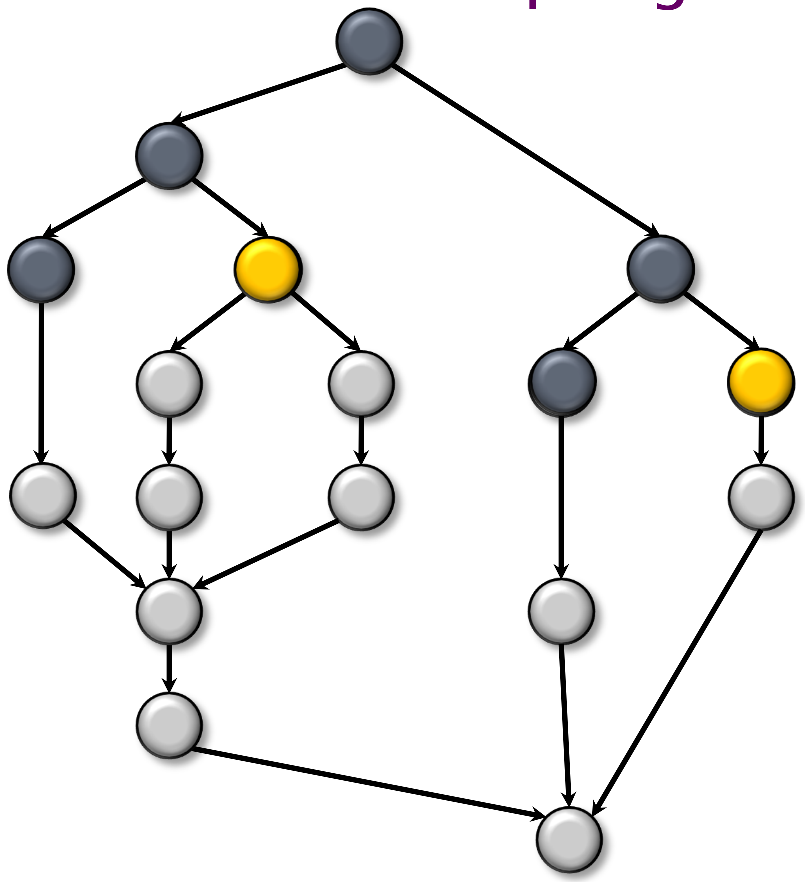


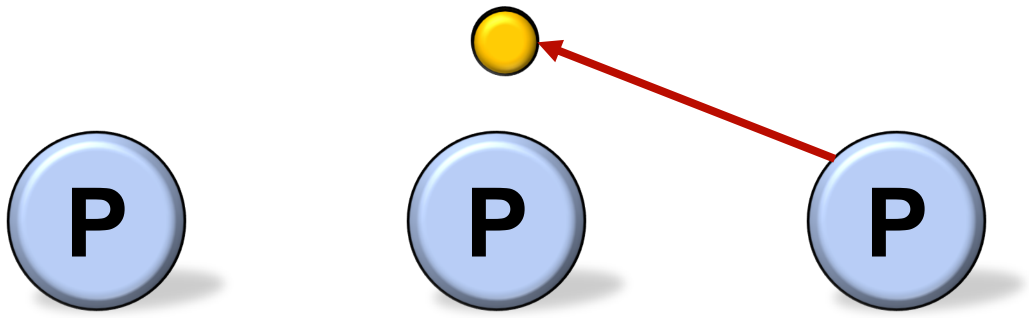
Overhead of work-stealing scheduler

Bound the number of steals (whp):

$$O(pD)$$

P = 3





Overhead of work-stealing scheduler

Bound the number of steals (whp):

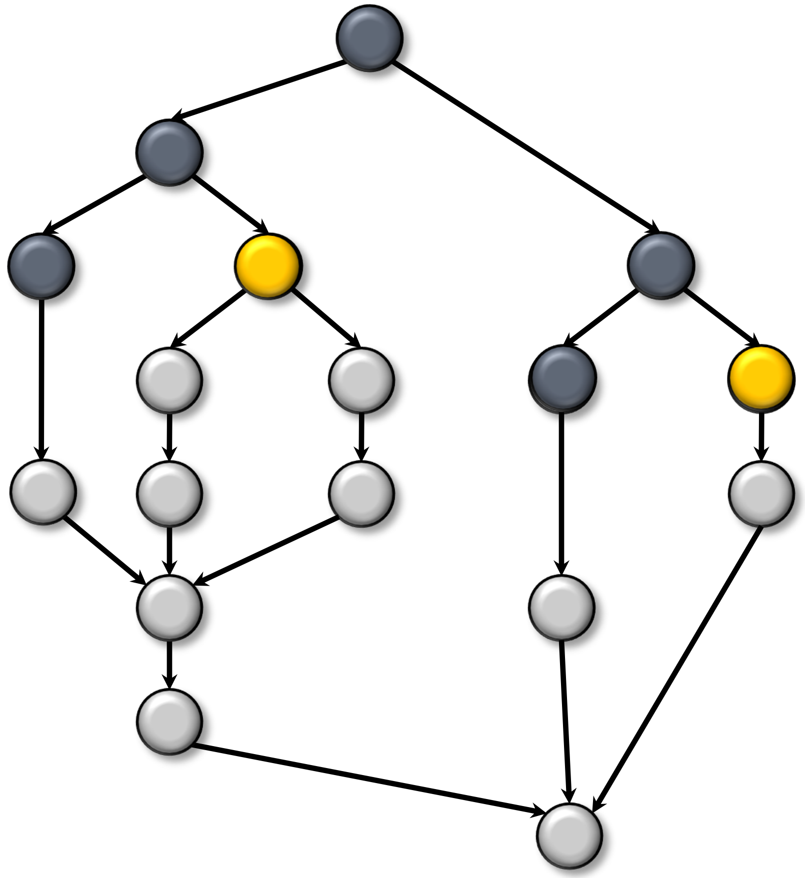
$$O(pD)$$

Running time (whp):

$$T = \frac{W + O(pD)}{p} = \frac{W}{p} + O(D)$$

Cache reload:

$$O(pD)$$

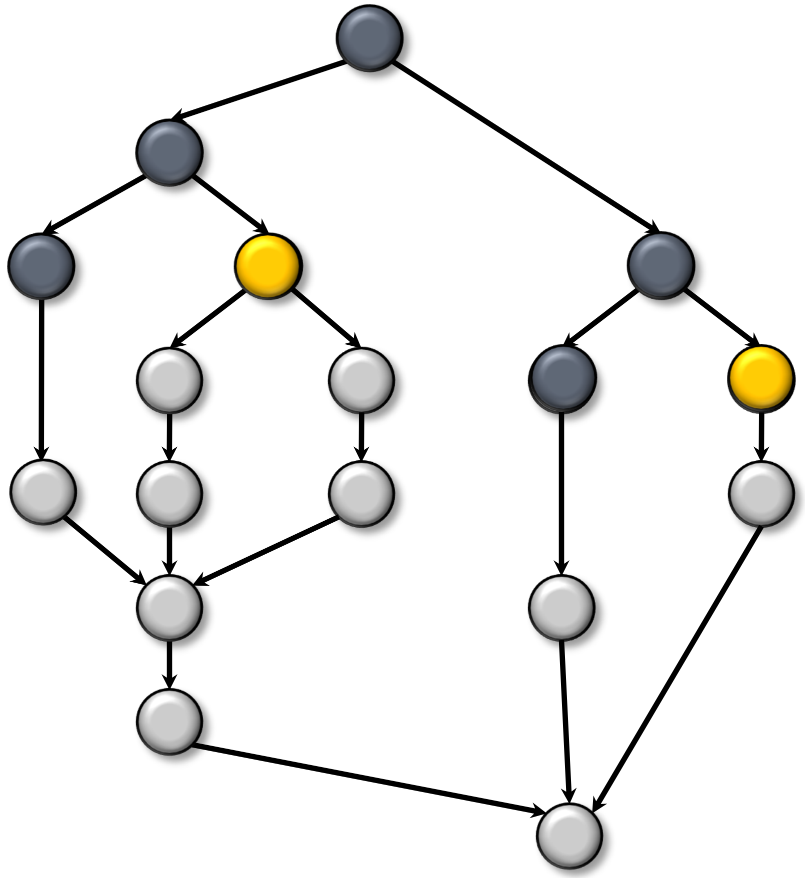


Assumptions

Steals come asynchronously

Multiple steals can be made to the same thread, and one wins (adversarially)

A successful steal from thread A would not block two consecutive steals from another thread B



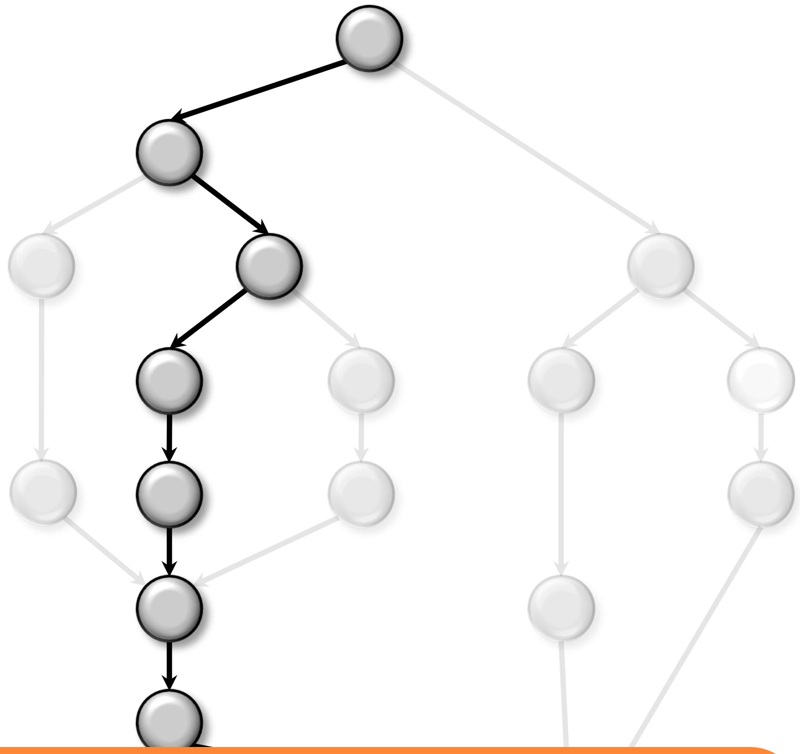
Proof outline

Consider one specific path

Left child: executed directly after the previous node

Right child:
- Stolen by another thread
- Executed when the current thread finishes the left side

Join node: executed when all previous nodes are finished



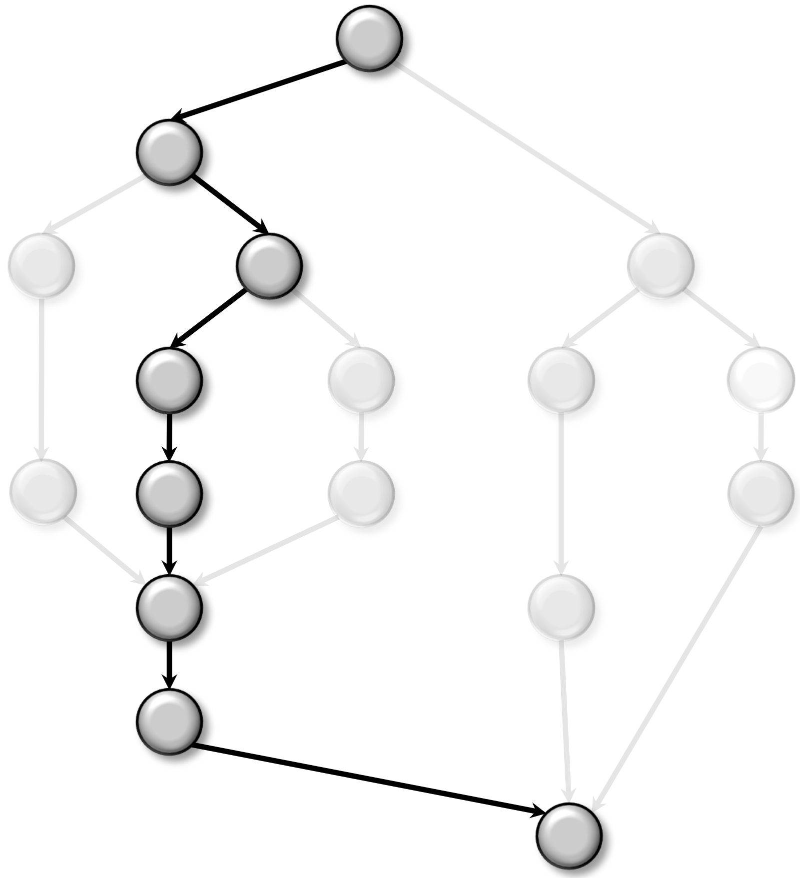
$$T = \frac{W + O(pD)}{p} = \frac{W}{p} + O(D)$$

Proof outline

Consider one specific path

Consider the worst case:

- All nodes are right child
- All of them need to be stolen



We want to show that $O(pD)$ steals are sufficient to steal D tasks *whp*

How many steals do we need?

Challenge: steals happen asynchronously

- They can block each other

Best case: steals are attempted one after another

Each steal has $1/(p-1)$ probability to steal one task

Chernoff bound: for n independent random variables in $\{0,1\}$, let X be the sum, and $\mu = \mathrm{E}[X]$, then for any $0 < \delta < 1$,

$$\Pr(X \geq (1-\delta)\mu) \leq e^{-\frac{\delta^2 \mu}{2}}$$

How many steals do we need?

Best case: steals are attempted one after another

Each steal has $1/(p-1)$ probability to steal one task

Let's say we have $2(p-1)(D+\ln(1/\epsilon))$ steal attempts

The probability that we have at least D successful steals from $2(p-1)(D+\ln(1/\epsilon))$ attempts is $1-\epsilon$

$$e^{-\frac{\delta^2\mu}{2}} = e^{-\frac{\left((\mu-D)/\mu\right)^2\mu}{2}} = e^{-\frac{(\mu-D)^2/\mu}{2}} < e^{(2D-\mu)/2} = e^{-\ln(1/\epsilon)} = \epsilon$$

$$\Pr(X \geq (1-\delta)\mu) \leq e^{-\frac{\delta^2\mu}{2}}$$

How many steals do we need?

Worst case: $p - 1$ steals are always attempted together

Probability that none of the steals touch the current thread:

$$\left(1 - \frac{1}{p-1}\right)^{p-1} < \frac{1}{e}$$

How many steals do we need?

Worst case: $p - 1$ steals are always attempted together

One task is stolen by probability at least $1 - 1/e$

Let's say we have $2e/(e-1)(D + \log(1/\epsilon))$ rounds of steals

Expected steals: $\mu = 2(D + \log(1/\epsilon))$

If we have less than D steals, then $\delta = (\mu - D)/\mu$, and

$$e^{-\frac{\delta^2 \mu}{2}} = e^{-\frac{((\mu-D)/\mu)^2 \mu}{2}} = e^{-\frac{(\mu-D)^2/\mu}{2}} < e^{(2D-\mu)/2} = \epsilon$$

The probability that we have at least D successful steals from $2(p-1)e(D + \ln(1/\epsilon))/(e-1)$ attempts is $1 - \epsilon$

How many steals do we need?

To get D steals with probability $1 - \epsilon$, we need

- Best case: $2(p-1)(D + \ln(1/\epsilon))$ steals
- Worst case: $2(p-1)(D + \ln(1/\epsilon))e/(e-1)$ steals

We want to guarantee probability with $1 - 1/n^c$

In a DAG with depth D, there are in total $\leq 2^D$ paths

Let $\epsilon = 1/(2^D \cdot n^c)$, then $\ln(1/\epsilon) = c \ln n + D \ln 2$

$O(pD)$ steals are sufficient for all possible paths *whp*

Overhead of work-stealing scheduler

The number of steals (whp):

$$O(pD)$$

Running time (whp):

$$T = \frac{W + O(pD)}{p} = \frac{W}{p} + O(D)$$

Cache reload:

$$O(pD)$$