Theoretically Efficient Parallel Graph Algorithms Can Be Fast And Scalable

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Joint work with Laxman Dhulipala and Guy Blelloch (CMU)

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Graphs are becoming very large

	Asymmetric	Symmetrized
	41 million vertices 1.5 billion edges (6.2 GB)	41 million vertices 2.4 billion edges (9.8 GB)
YAHOO!	1.4 billion vertices 6.6 billion edges (38 GB)	1.4 billion vertices 12.9 billion edges (63 GB)
Common Crawl	3.5 billion vertices 128 billion edges (540 GB)	3.5 billion vertices 225 billion edges (928 GB)

- Need efficient graph processing to do analytics in a timely fashion

Large-Scale Graph Processing

- Write algorithms for large distributed clusters or supercomputer
- Prior results on Common Crawl graph (225B edges):

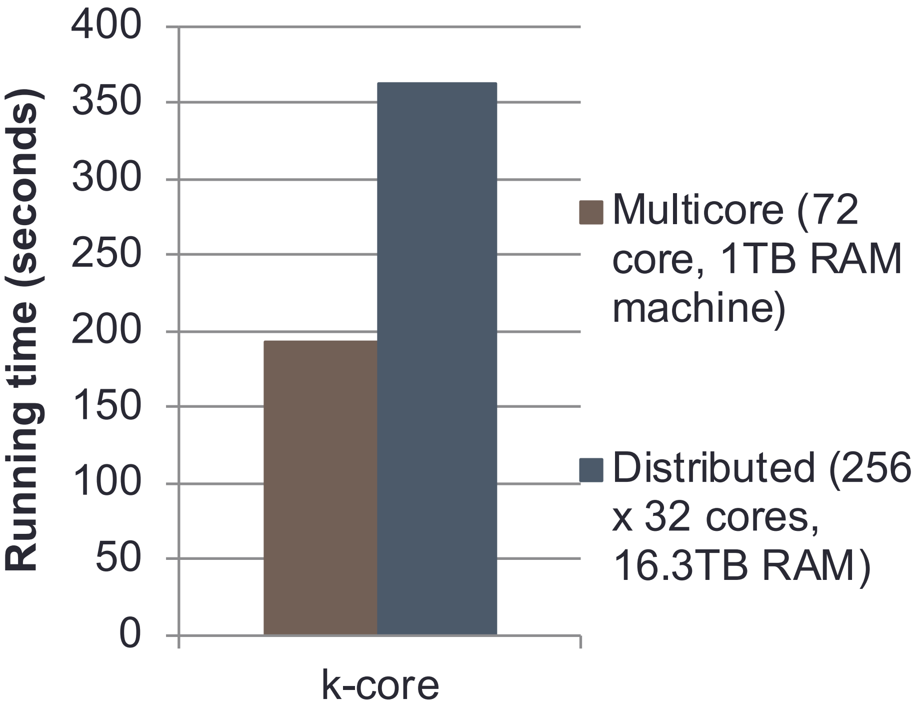
Distributed Algorithms	Hardware	Running Time
Approx. k-core (Slota et al.)	256 x 32 cores, 16.3TB RAM	363 sec
Largest Conn. Comp. (Slota et al.)	256 x 32 cores, 16.3TB RAM	63 sec
Conn. Comp. (Stergiou et al.)	1000 x 24 cores, 128TB RAM	341 sec

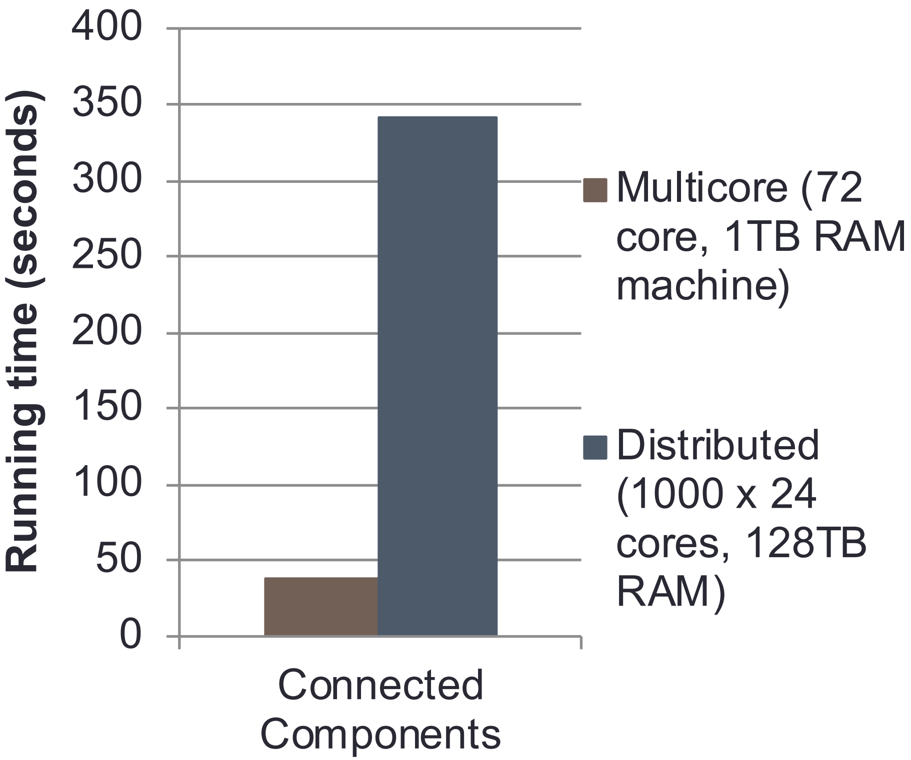
- Write algorithms for limited-memory machine that stream graphs from SSDs (TurboGraph, Mosaic, BigSparse)
  - Usually (up to an order of magnitude) slower but much more cost-efficient

What about in-memory computation on a single machine with 1TB RAM?

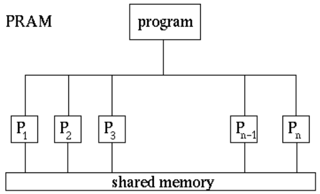
Multicore Results

- Results on Common Crawl graph (3.5B vertices, 225B edges)

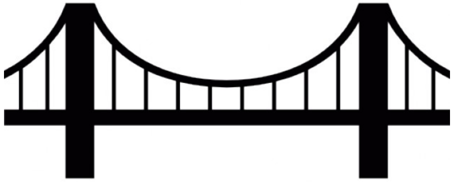


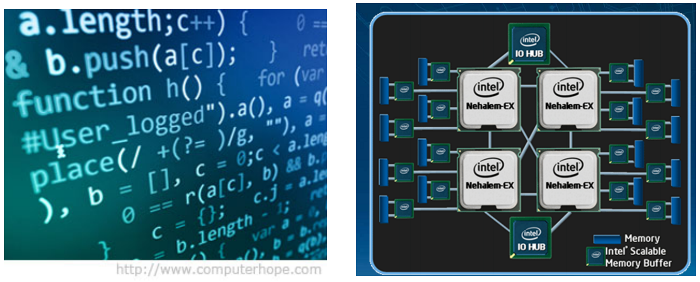


Theoretically-Efficient Practical Algorithms

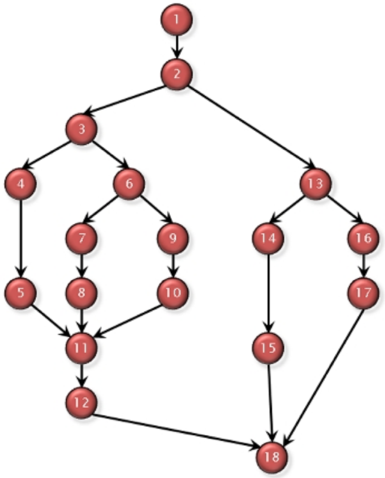


O(n log n)
O(n)
O(log n)





- Want good performance under many different settings, e.g., different machines and larger datasets

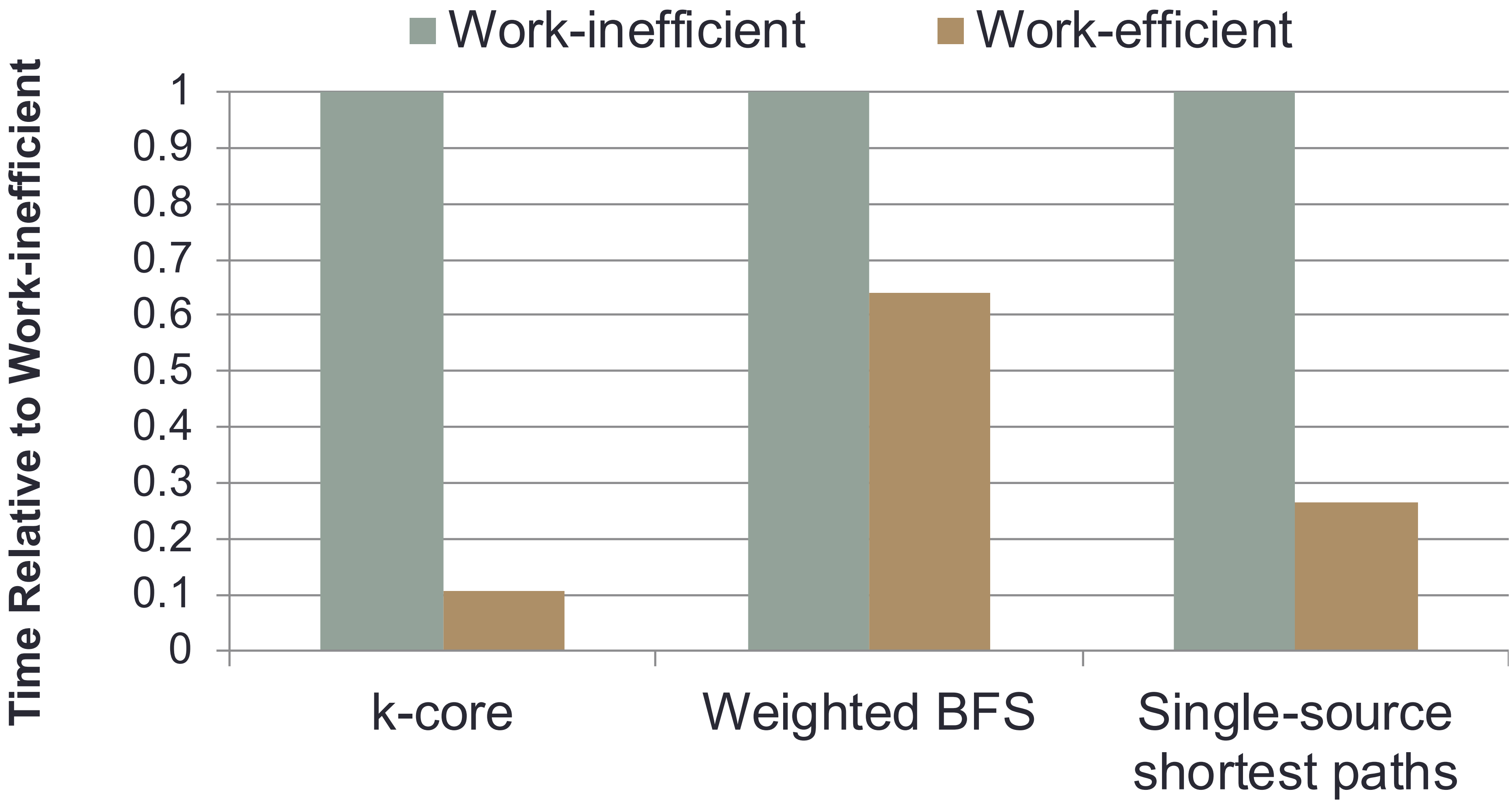


Work *= number of operations*
Depth *= length of longest sequential dependence*

Running time *≤ (Work/#processors) + Depth*

- Goal: Minimize depth without increasing work over best sequential algorithm (**work-efficient**)

Theoretically-Efficient Practical Algorithms



Theoretically-efficient graph algorithms can be fast

Contributions

- Theoretically-efficient parallel graph algorithms that are practical

Breadth-first search
Betweenness centrality
Connected components
Biconnected components
Triangle counting
k-core decomposition
Maximal independent set
Approximate set cover
Weighted BFS
Single-source shortest paths
Low-diameter decomposition
Strongly connected components
Minimum spanning tree
Maximal matching
Graph coloring

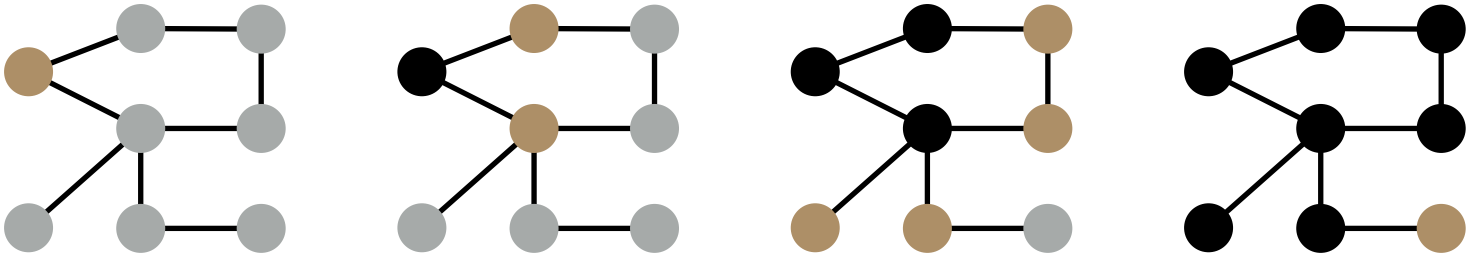
- Extended Ligra framework to support bucketing algorithms
- Theoretically-efficient optimizations
- Experimental evaluation on the largest publicly-available real-world graphs, outperforming existing results

Ligra: Frontier-Based Algorithms

Primitives

- Frontier data-structure (VertexSubset)
- Map over vertices in a frontier (VertexMap)
- Map over out-edges of a frontier (EdgeMap)

Example: Breadth-First Search



Round 1 Round 2 Round 3 Round 4

🟤 : in frontier ⚪ : unvisited ⚫ : visited

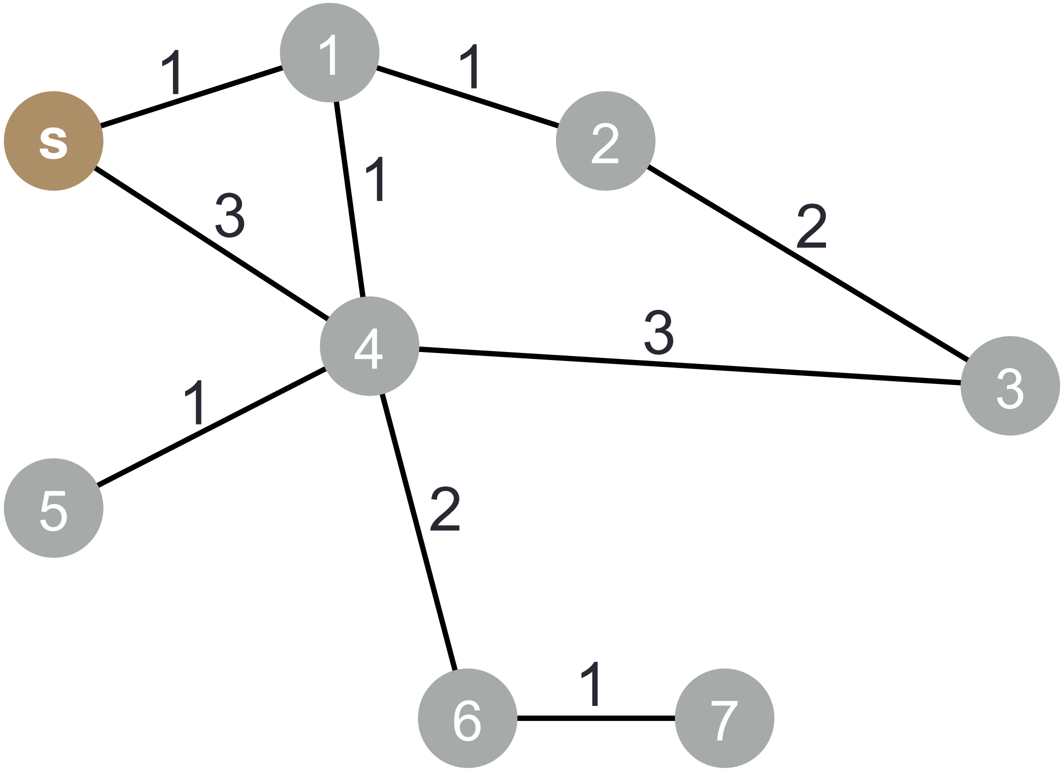
Some useful graph algorithms cannot be efficiently implemented in frontier-based frameworks

Example: Weighted Breadth-First Search

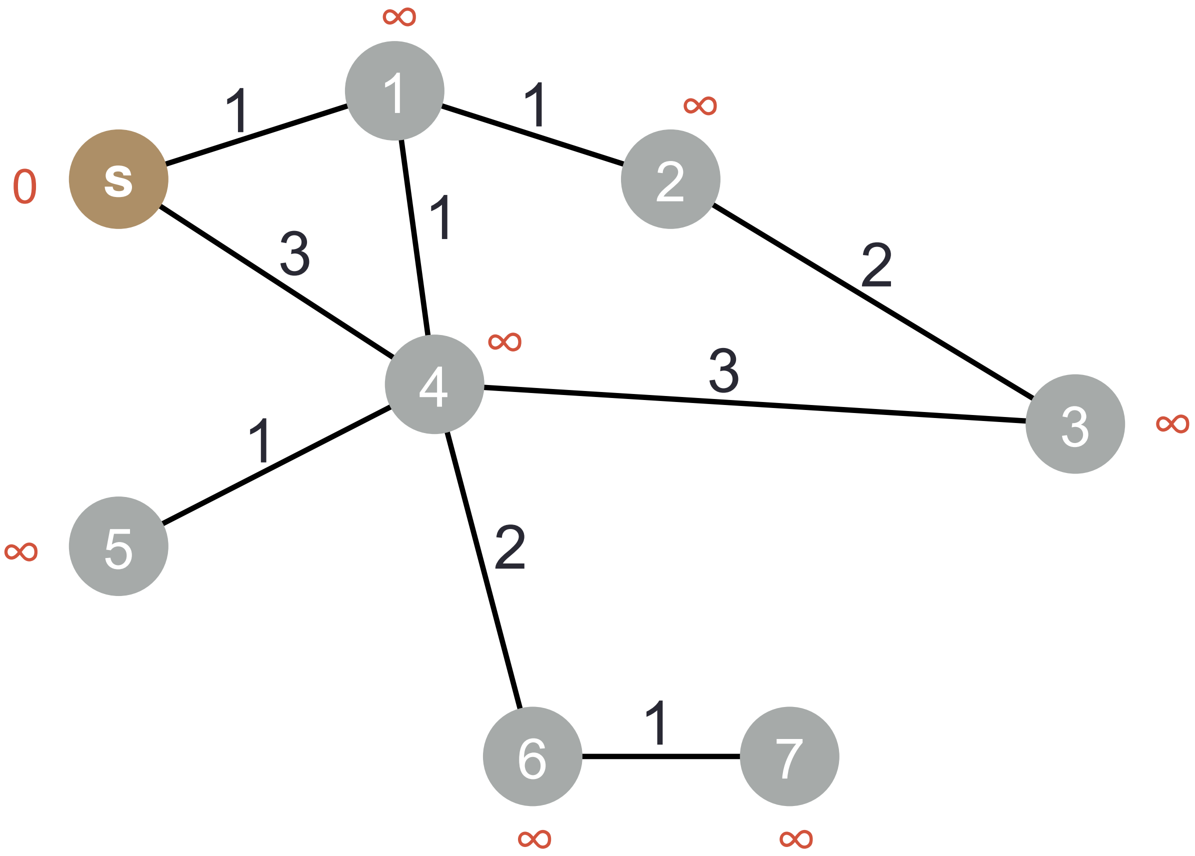
Given: $G = (V, E, w)$ with *positive integer edge weights*, $\mathbf{s} \subseteq V$

Problem: Compute the shortest path distances from **s**

Frontier-based approach: On each step, update distances of neighbors, place neighbors whose distance decreased onto next frontier



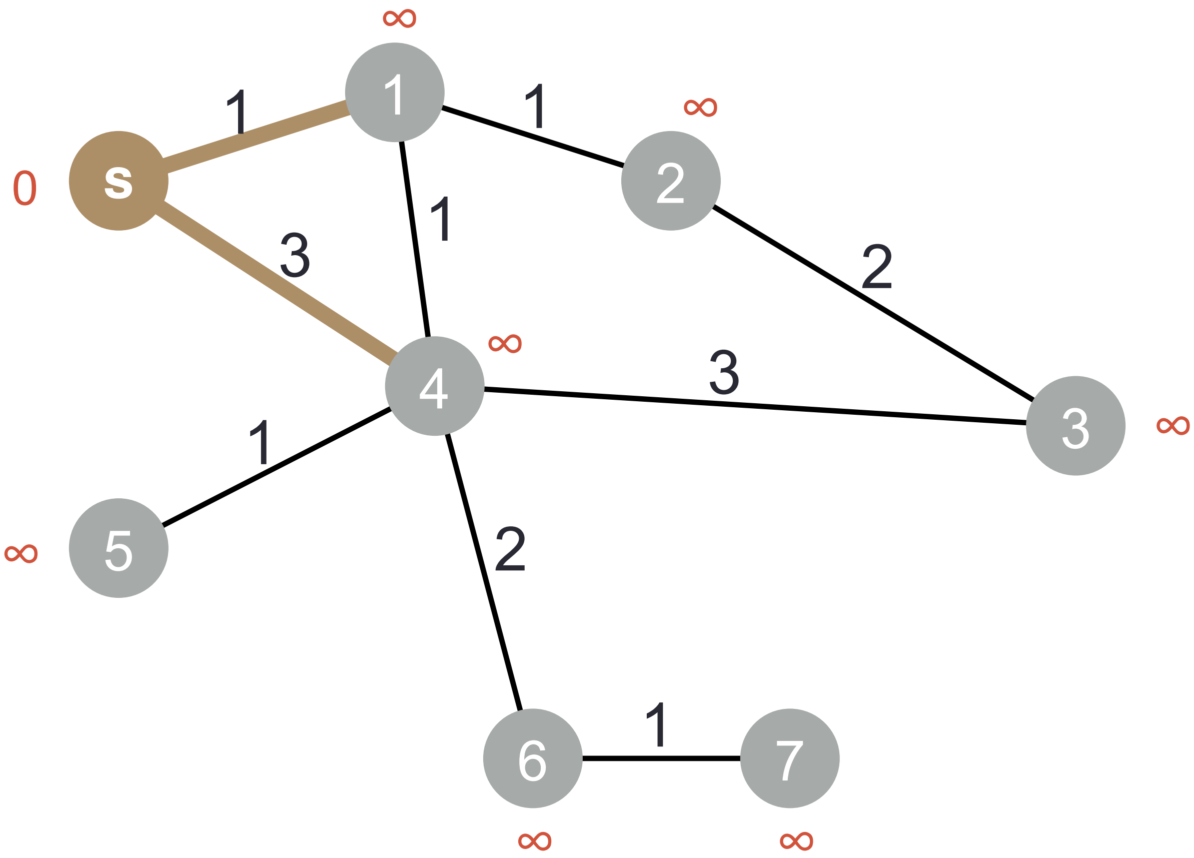
Example: Weighted Breadth-First Search



Frontier: s

Round 1

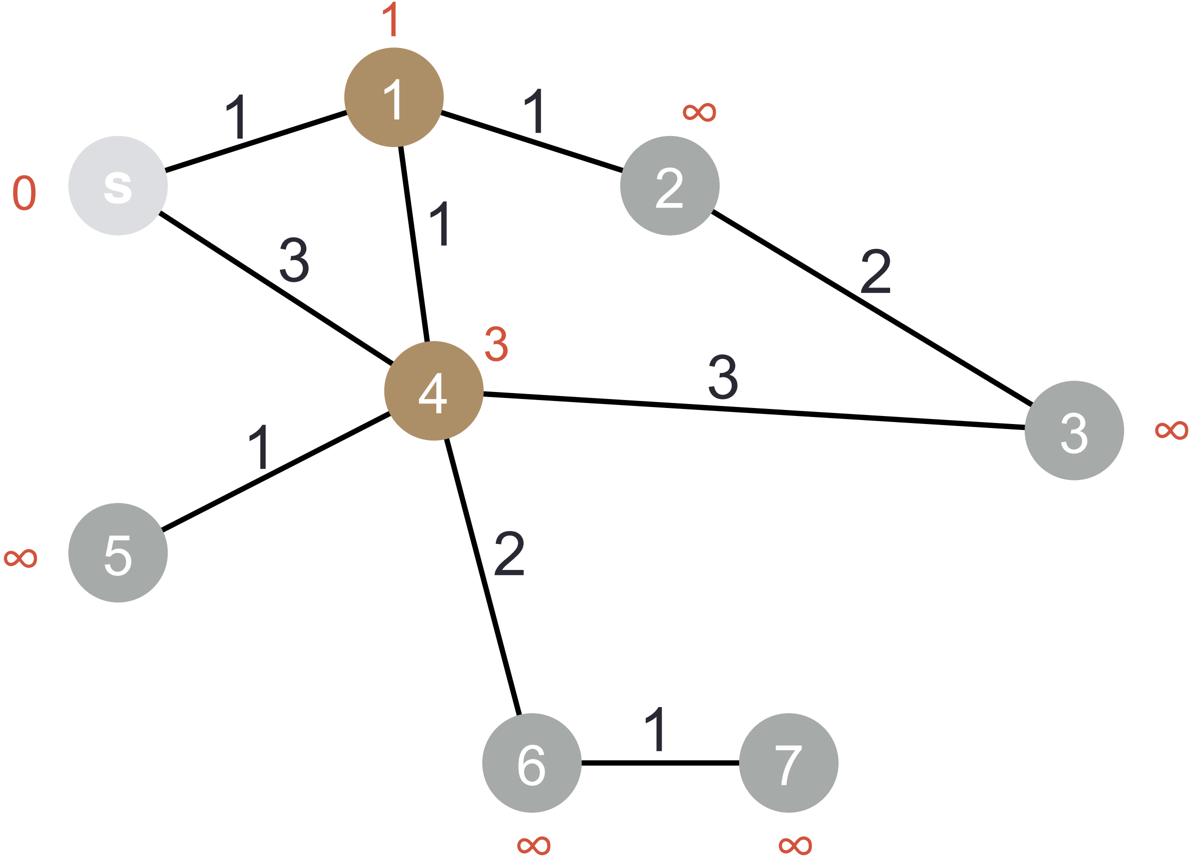
Example: Weighted Breadth-First Search



Frontier: s

Round 1

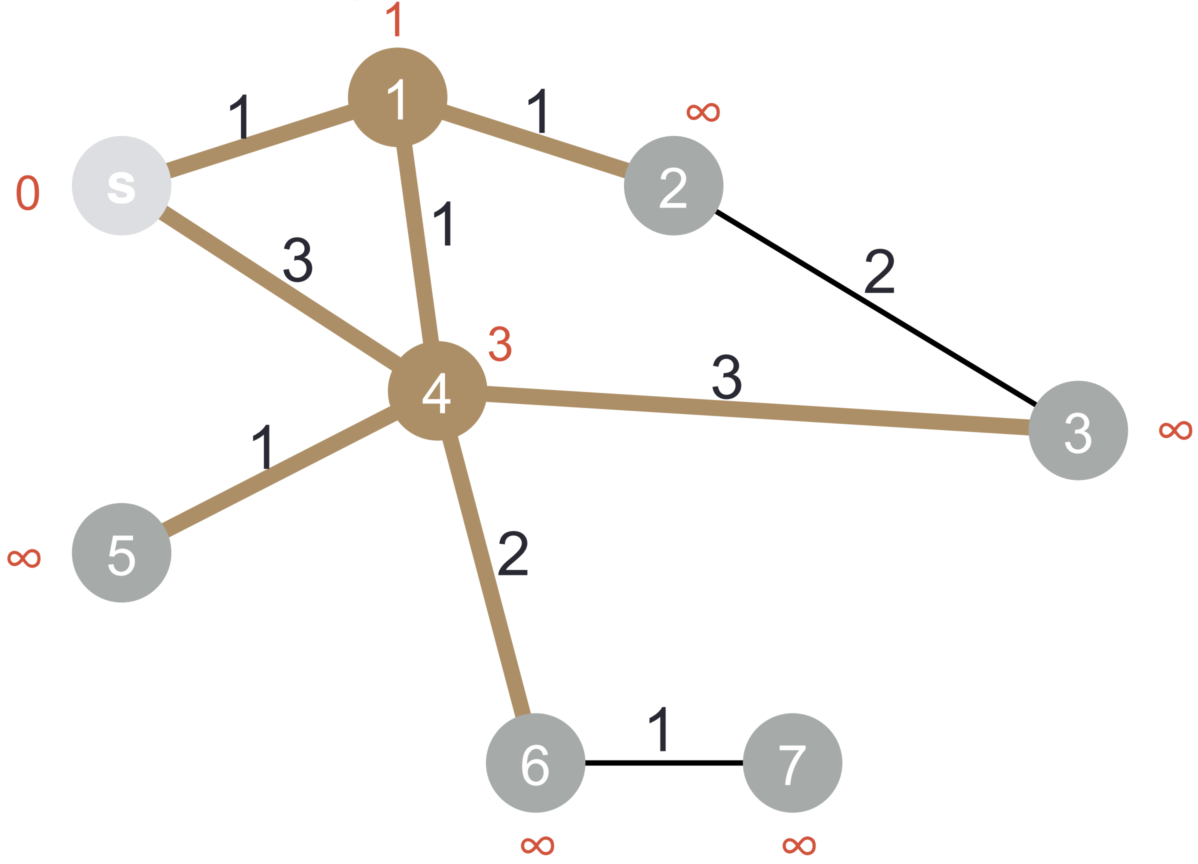
Example: Weighted Breadth-First Search



Frontier: 1 4

Round 2

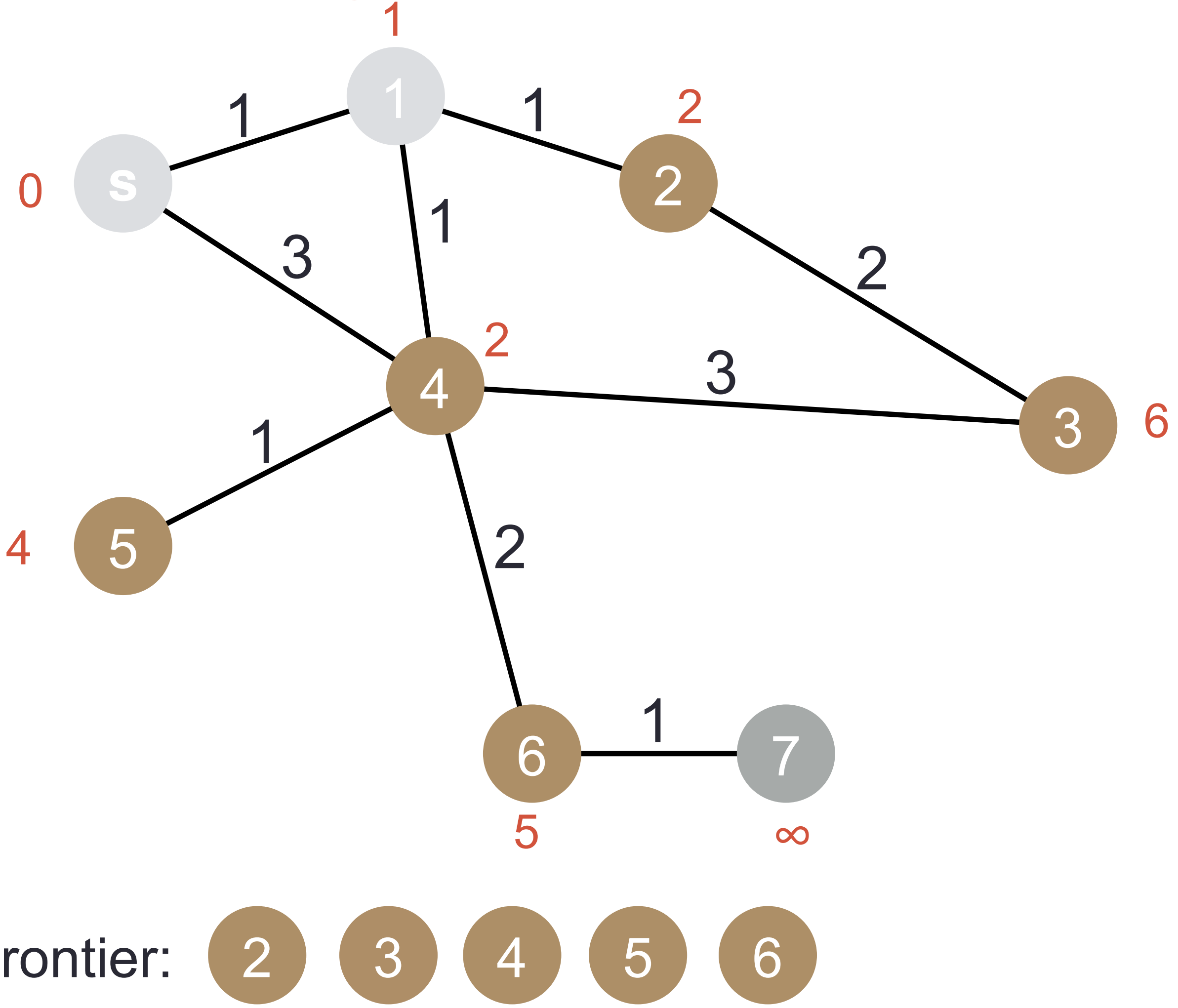
Example: Weighted Breadth-First Search



Frontier:

Round 2

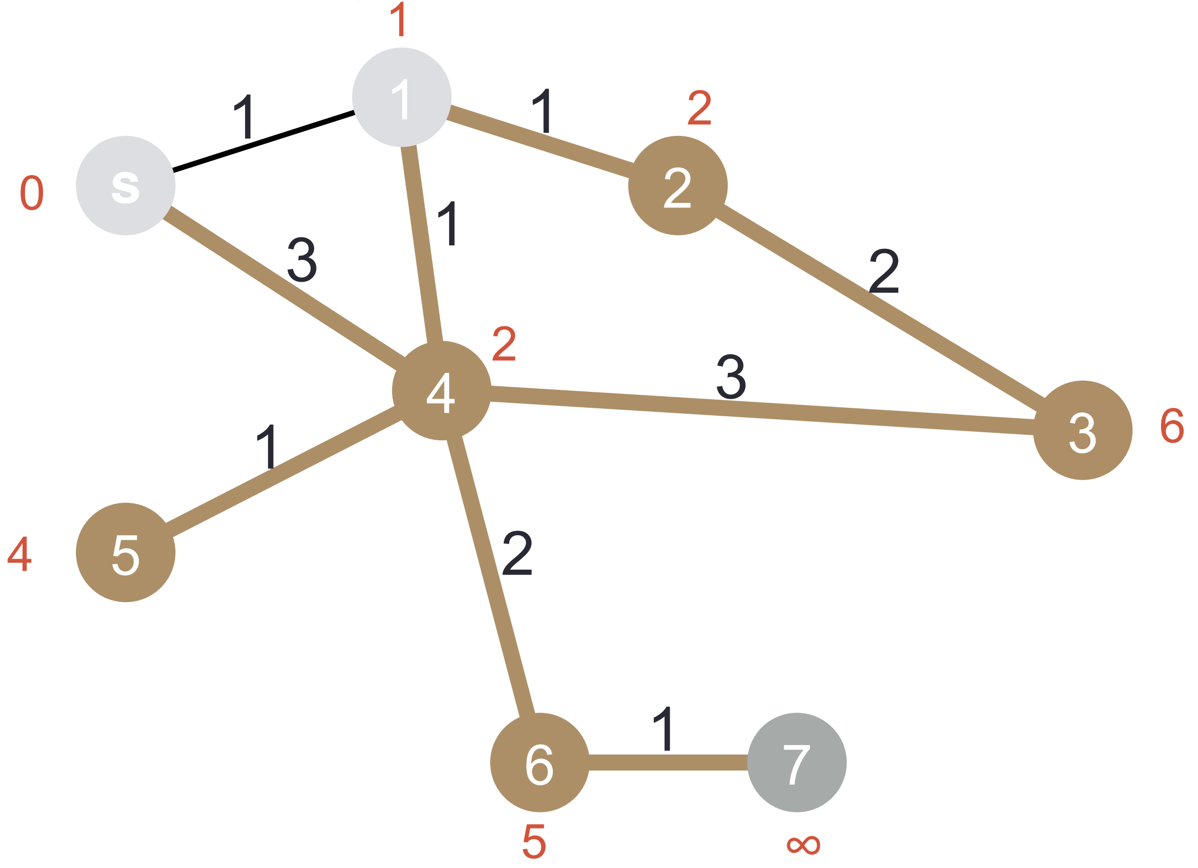
Example: Weighted Breadth-First Search



Frontier: 2 3 4 5 6

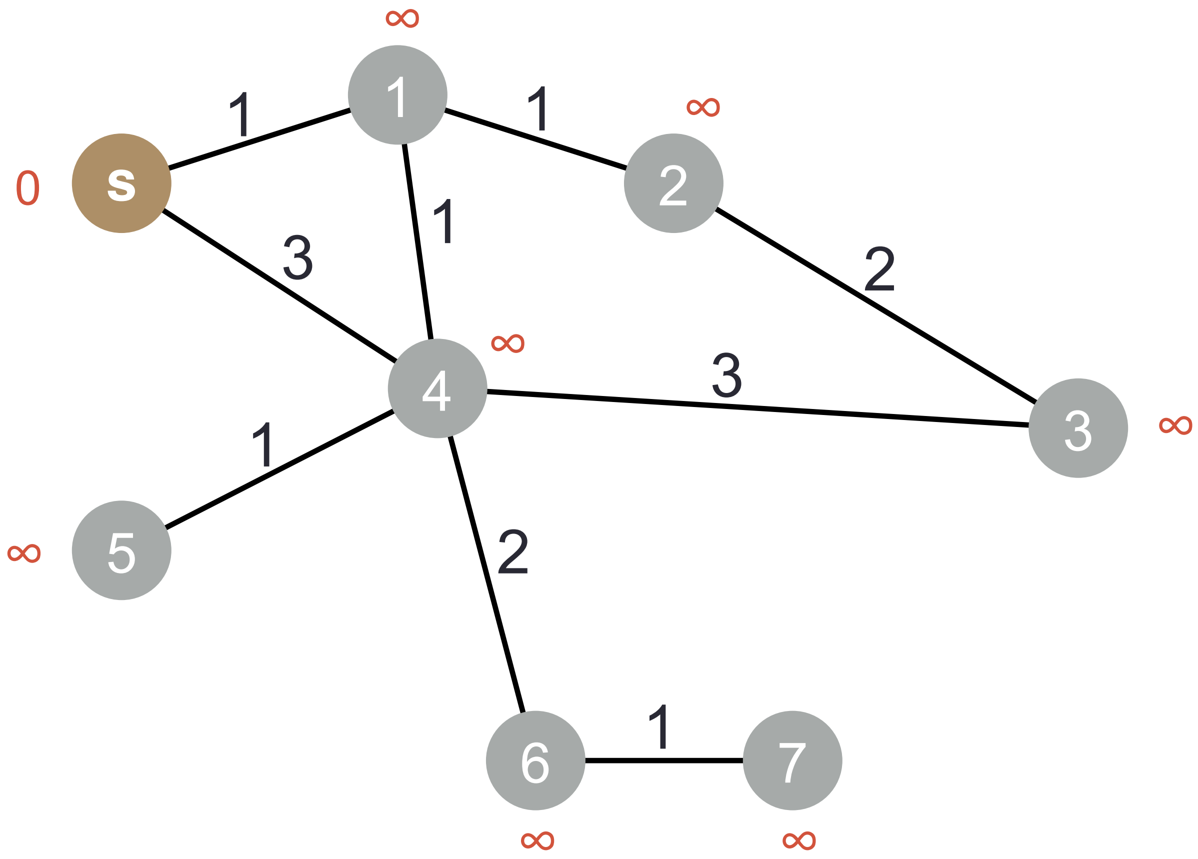
Round 3

Example: Weighted Breadth-First Search



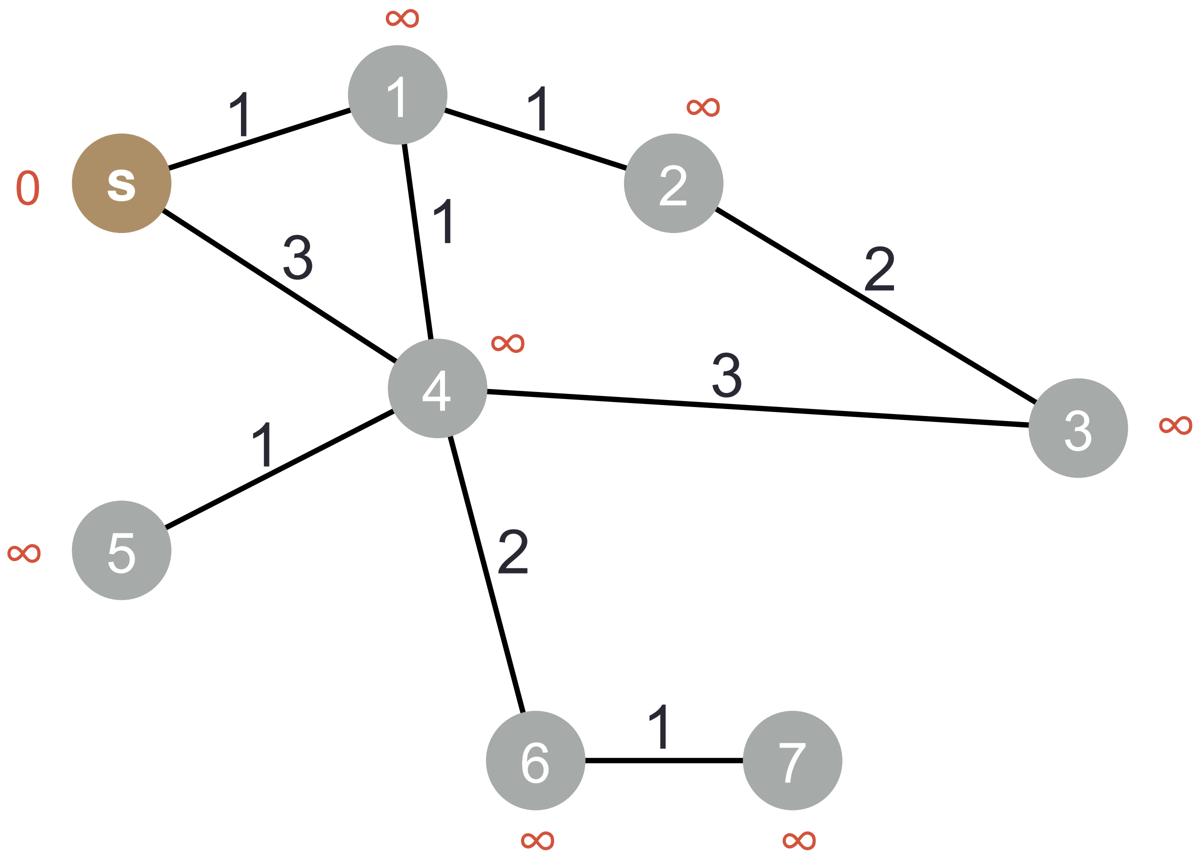
Takes O(VE) work, which is not work-efficient!

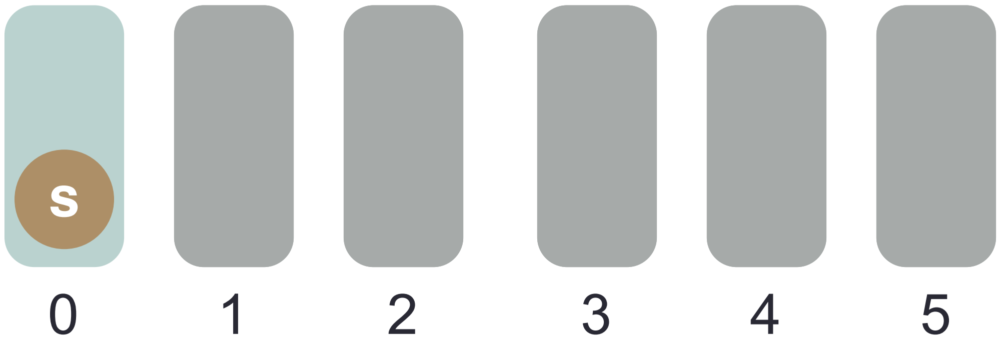
Sequential Weighted BFS



- Sequential algorithm runs in $O(D + |E|)$ work
- Run Dijkstra's algorithm, but use *buckets* instead of a priority queue
- Represent buckets using dynamic arrays

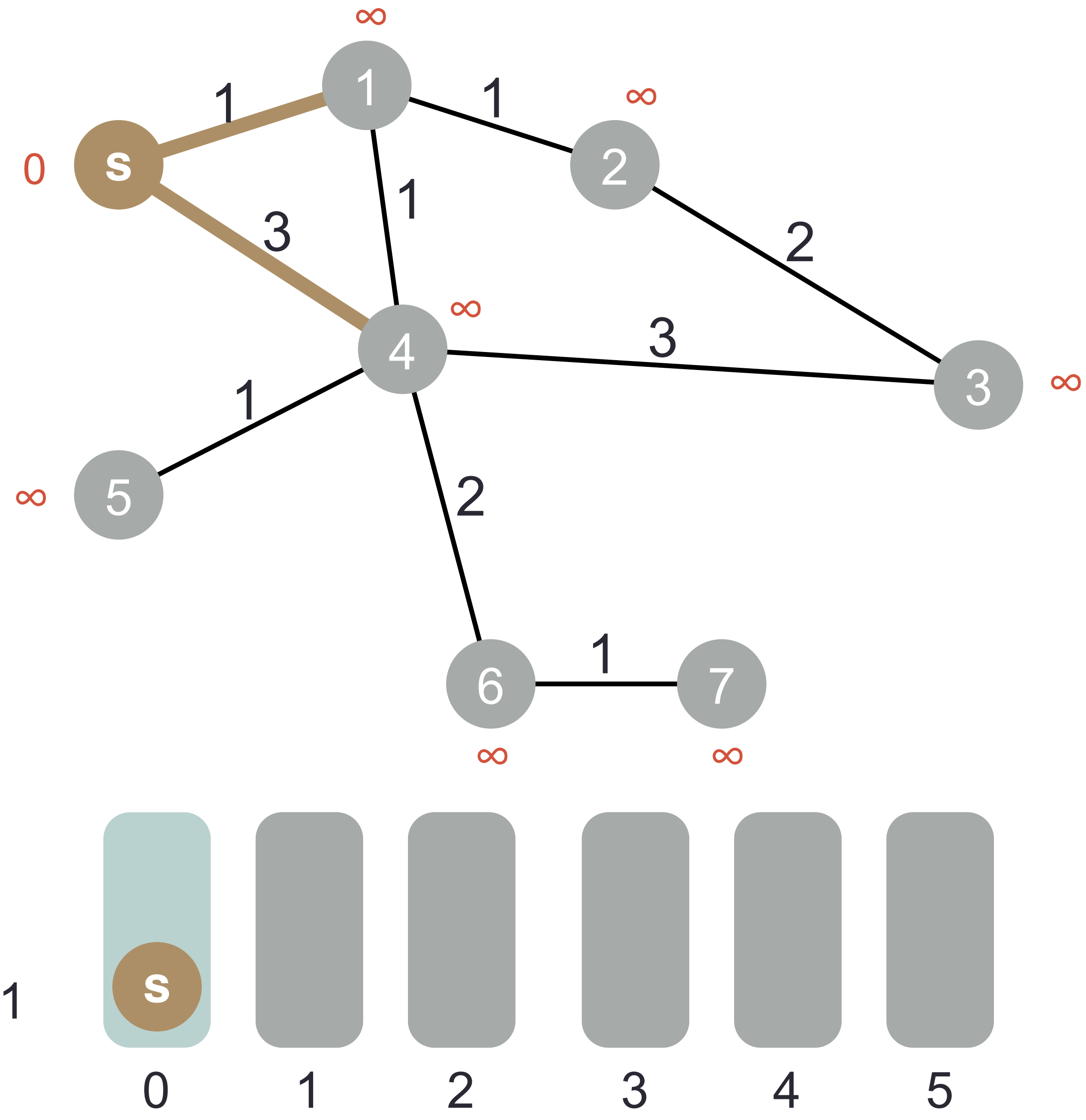
Sequential Weighted BFS



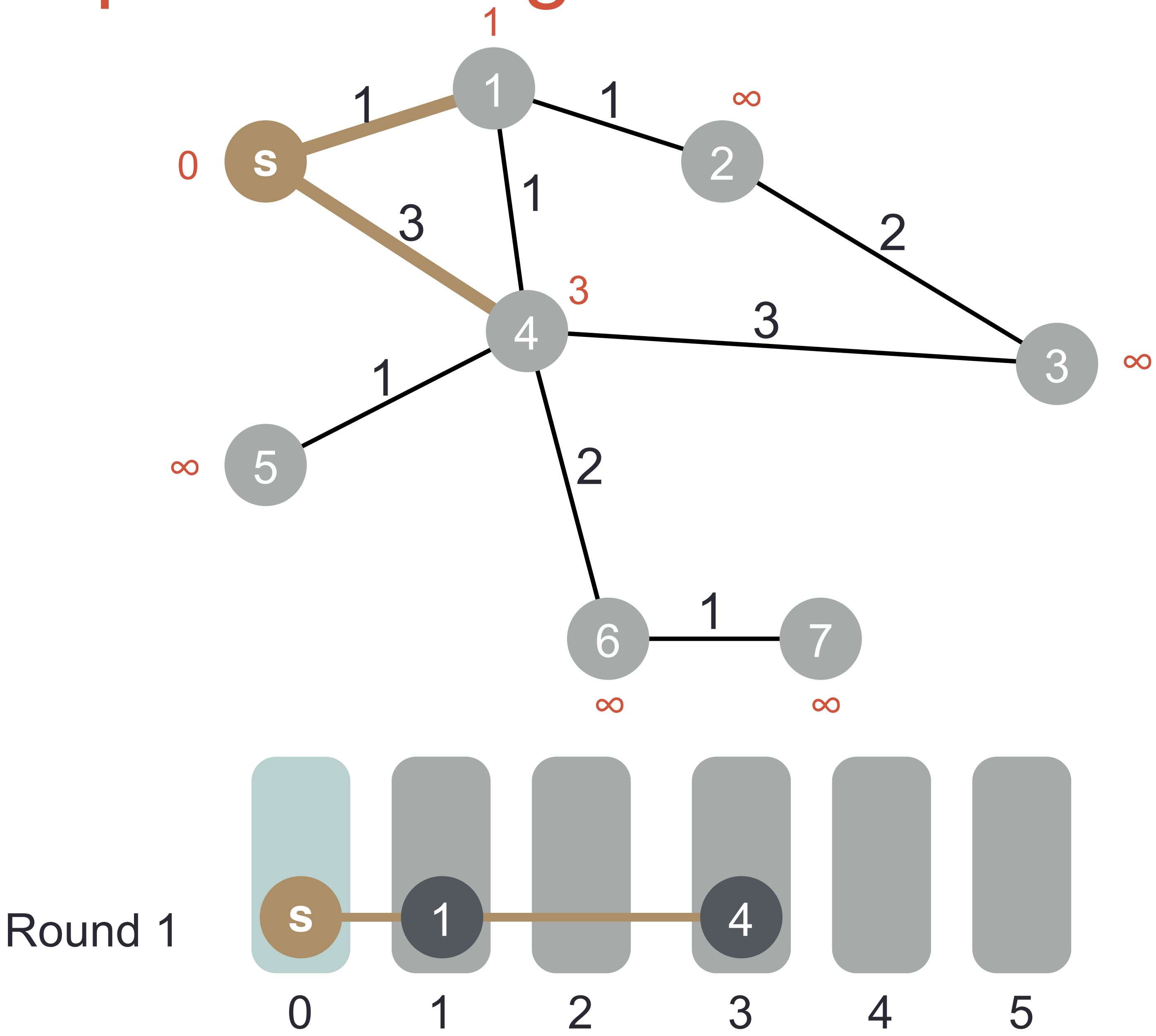


Round 1

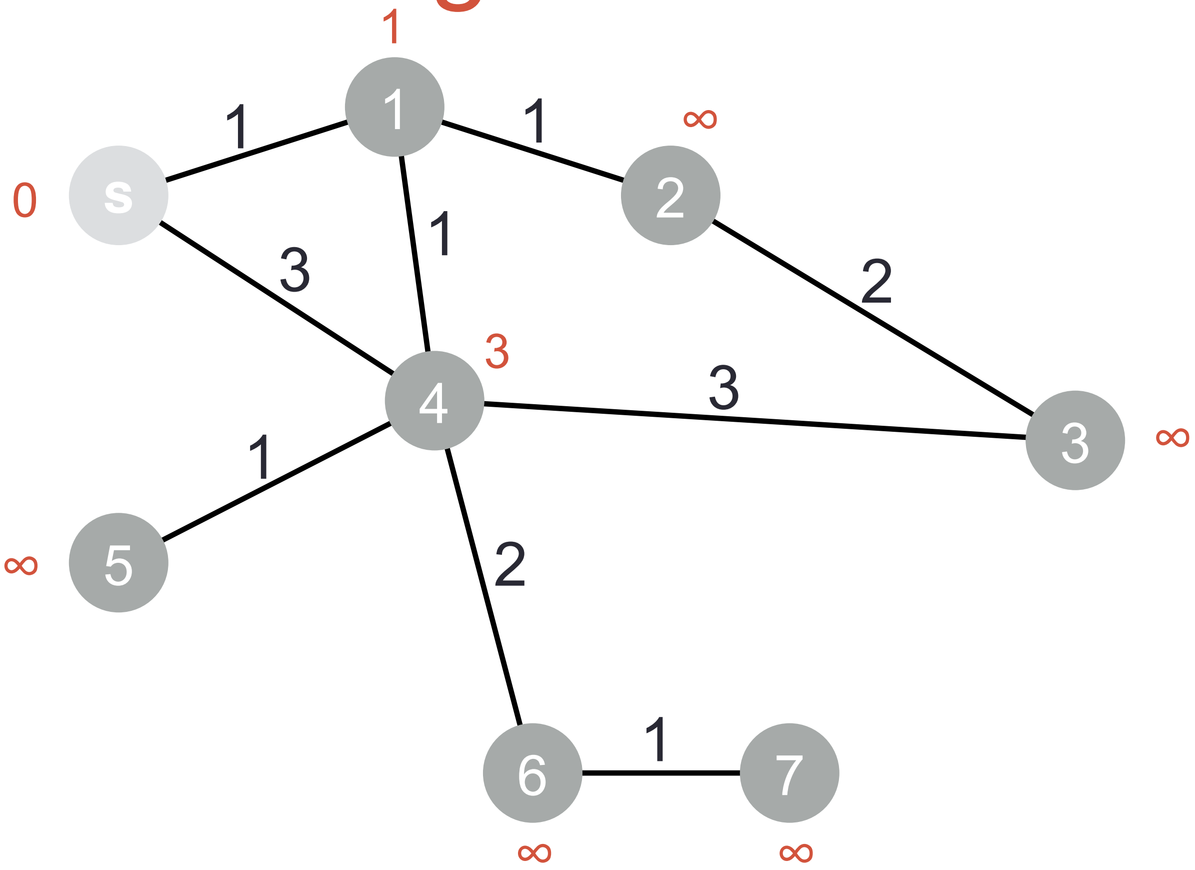
Sequential Weighted BFS



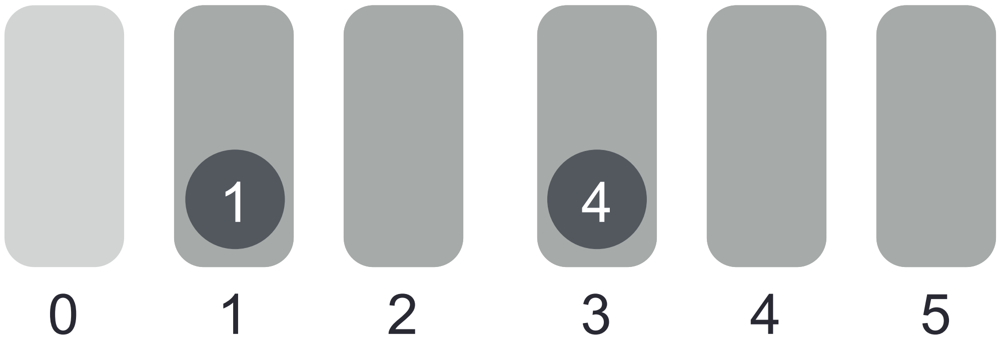
Sequential Weighted BFS



Sequential Weighted BFS



Round 1



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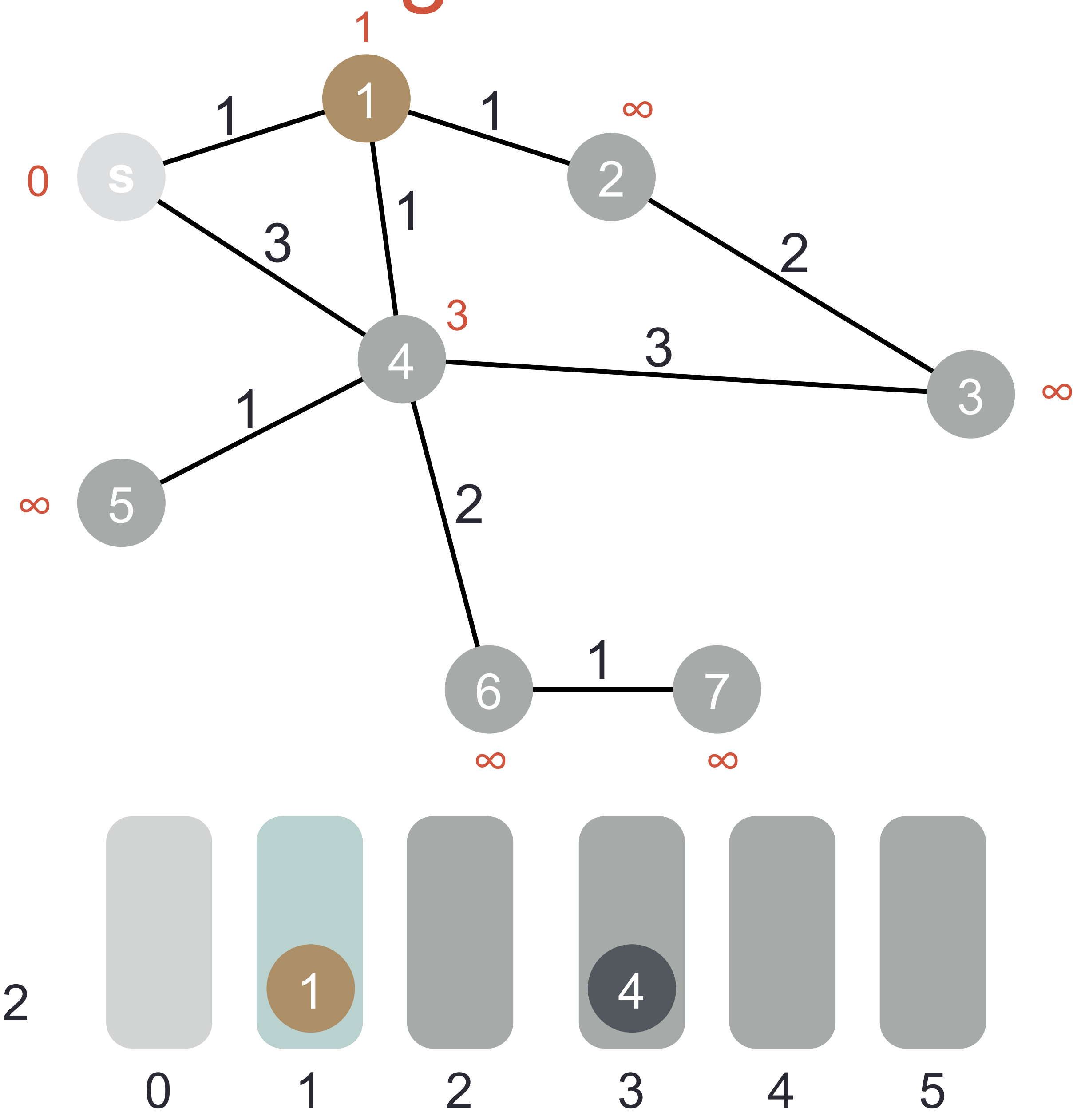

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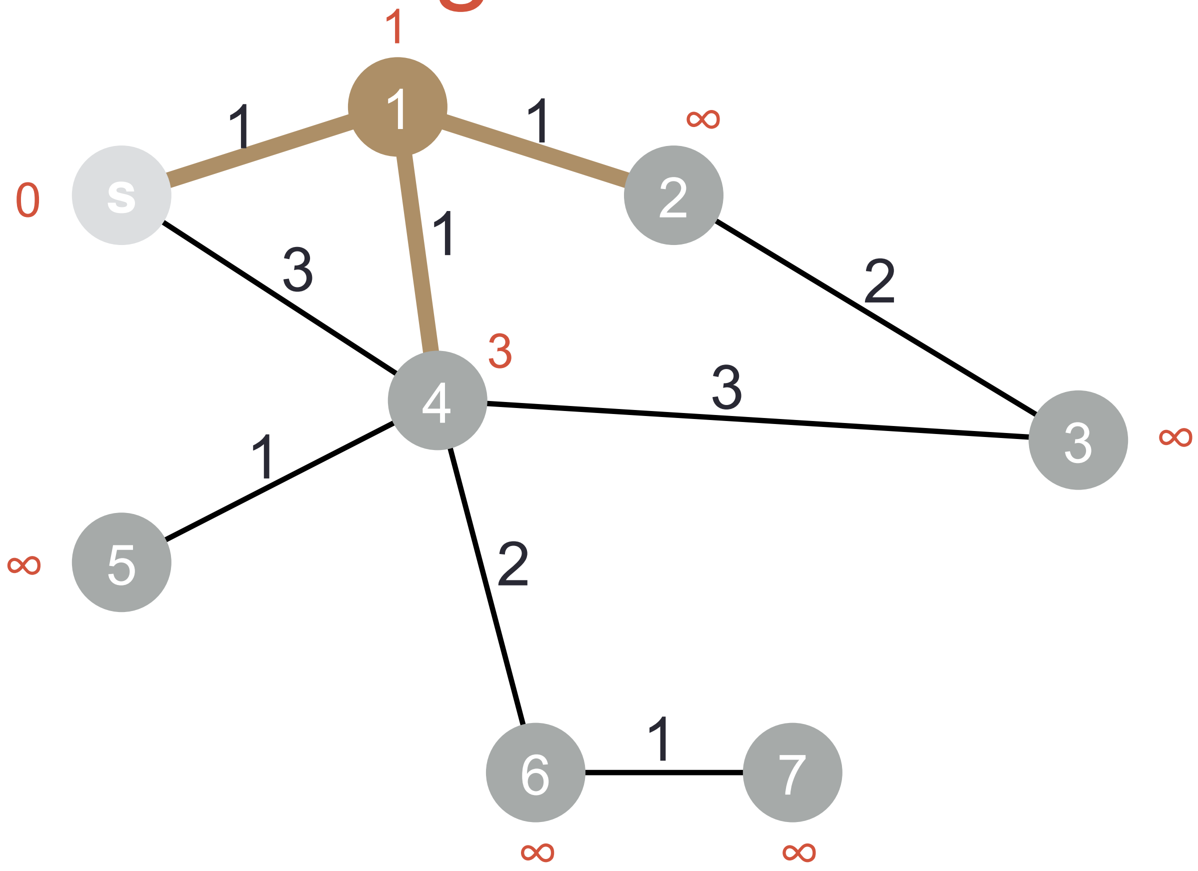
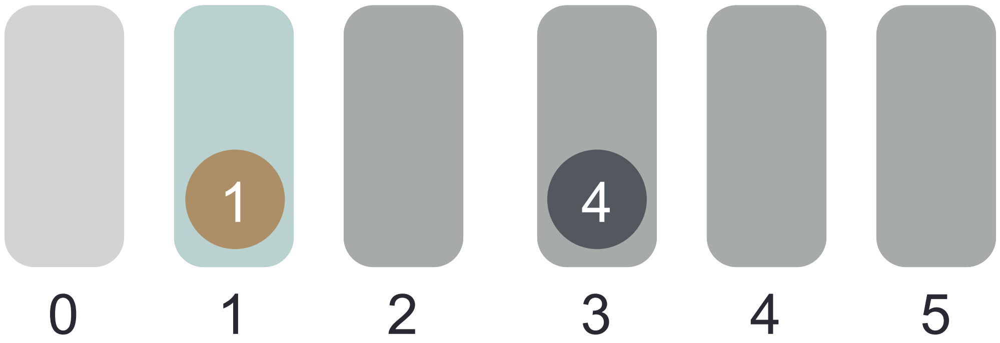
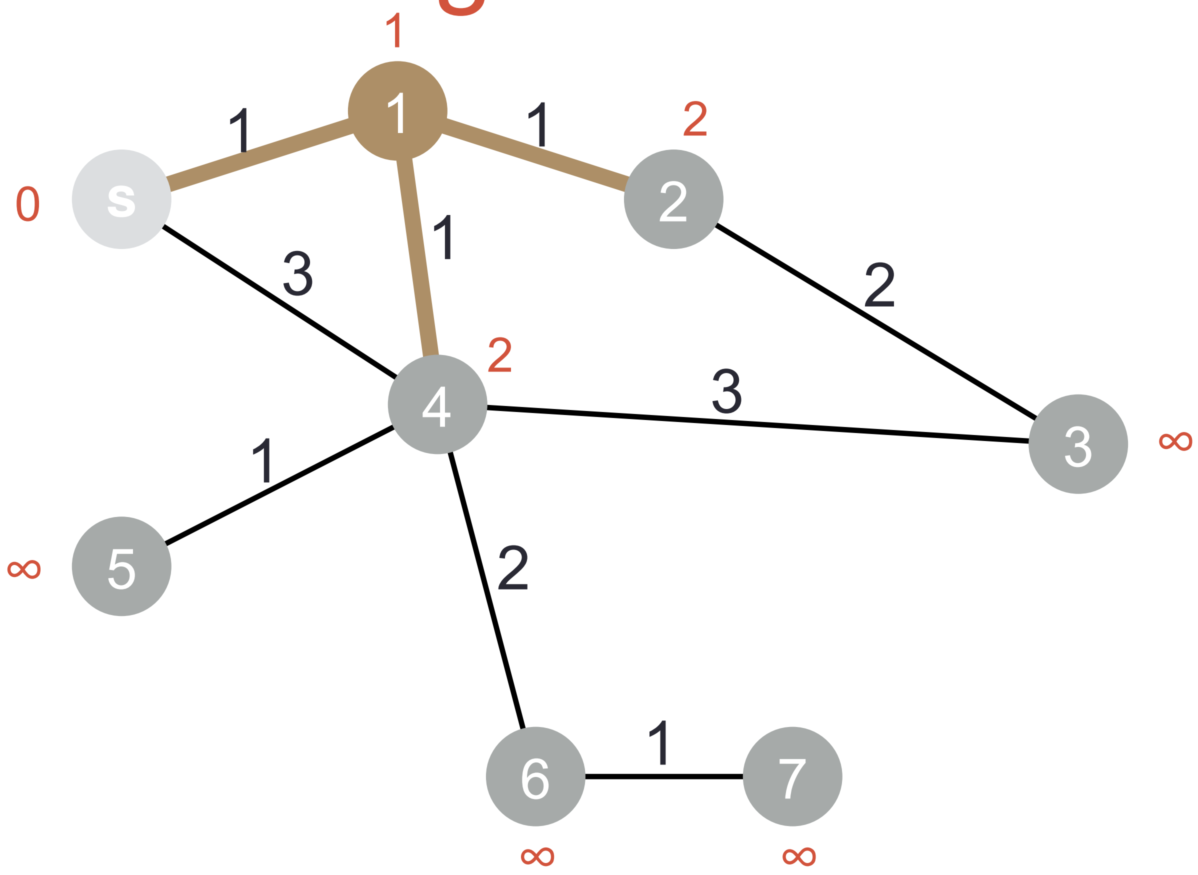
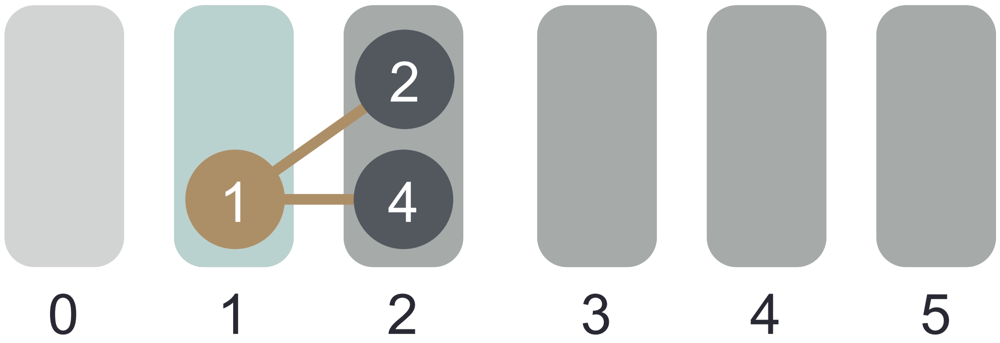




Round 2

Sequential Weighted BFS





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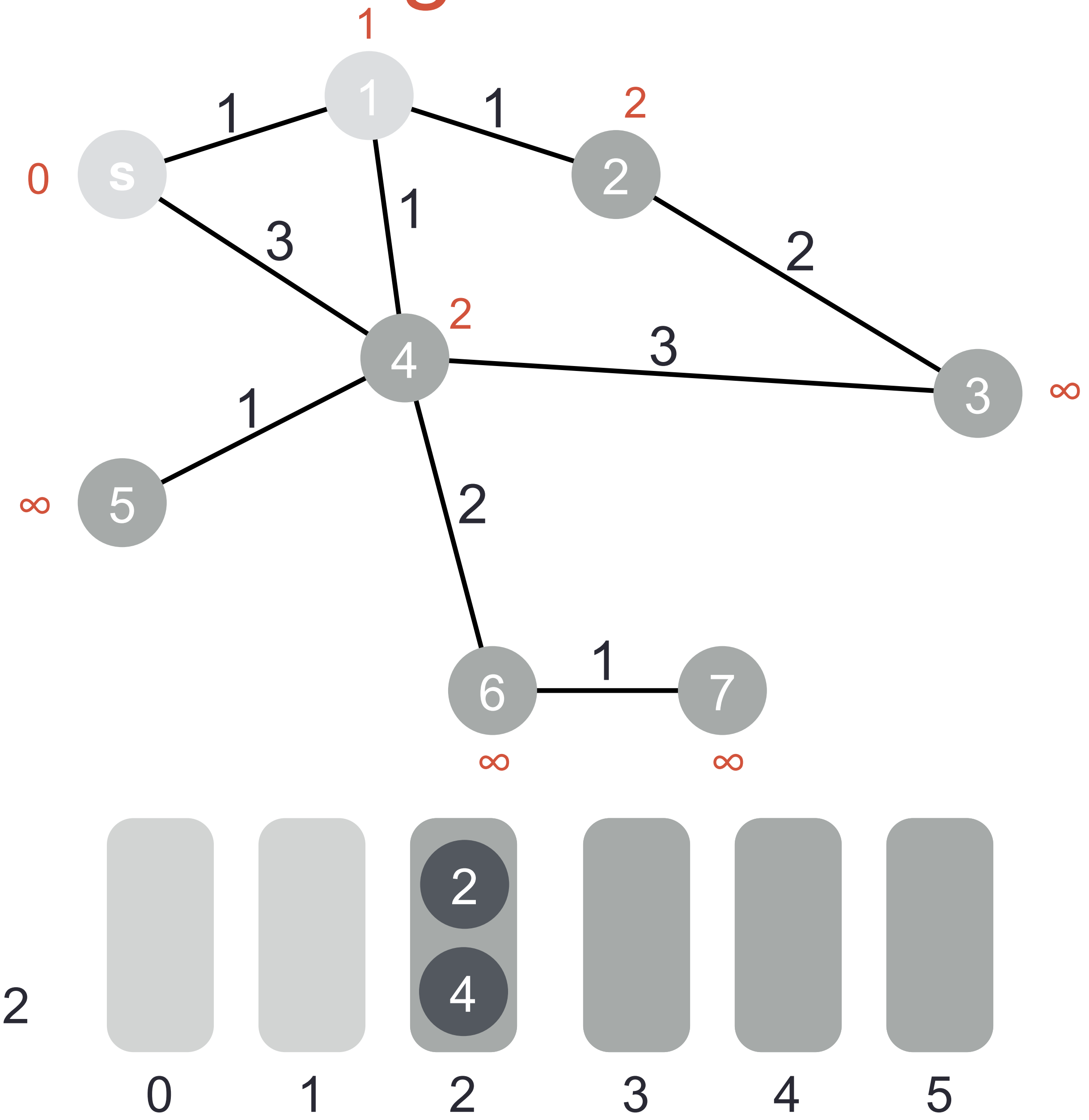
Sequential Weighted BFS



Round 2

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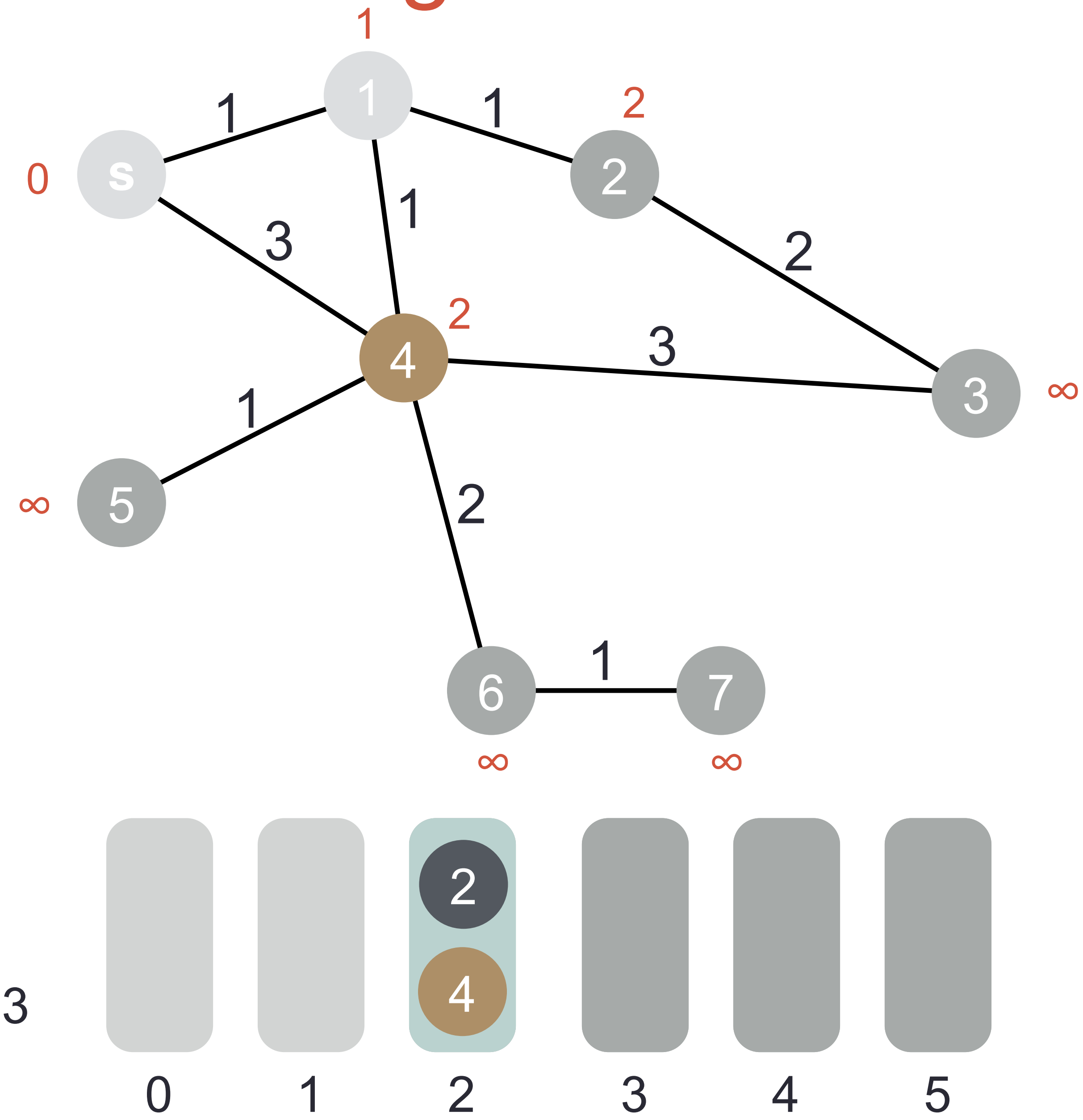

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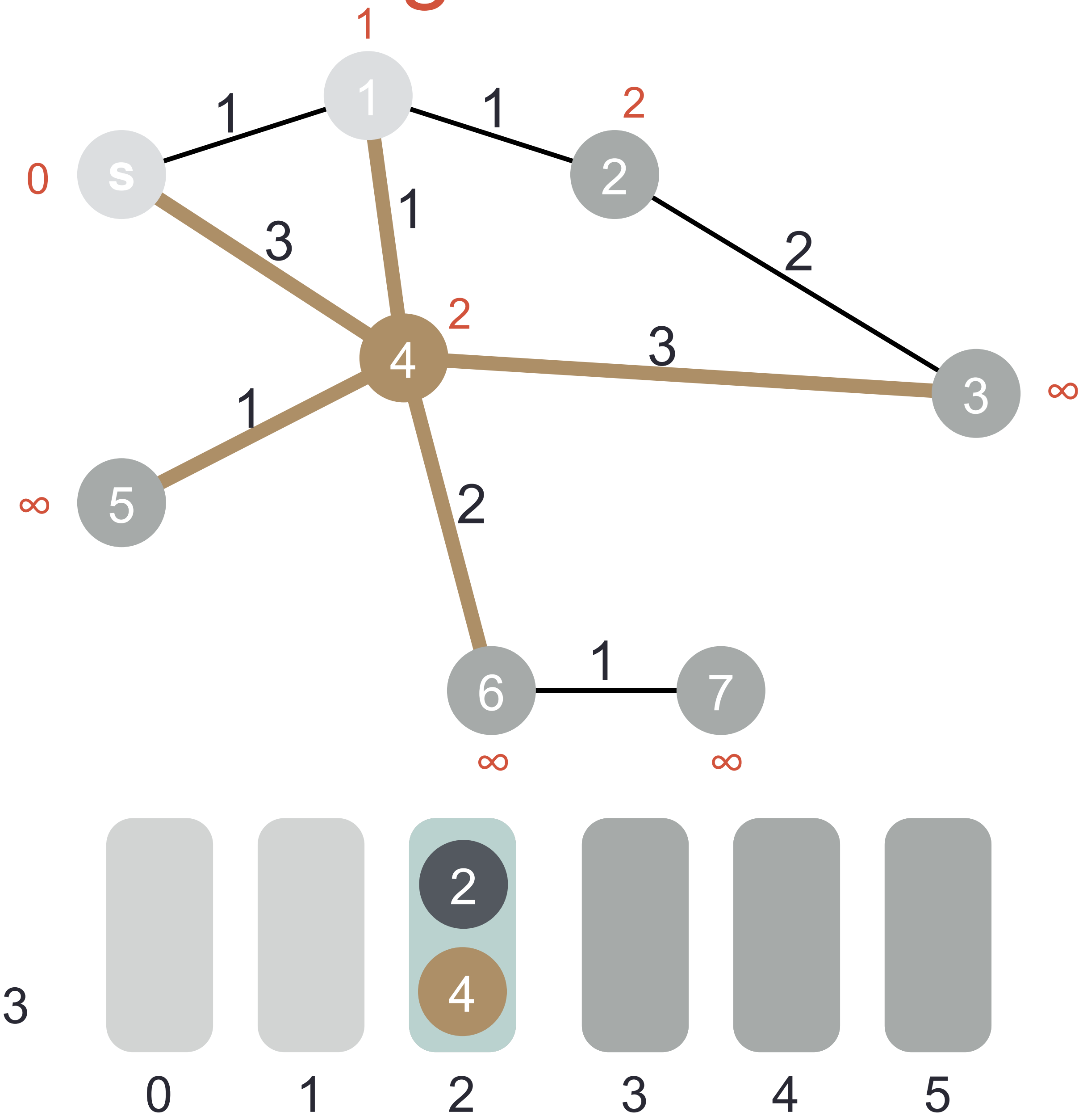
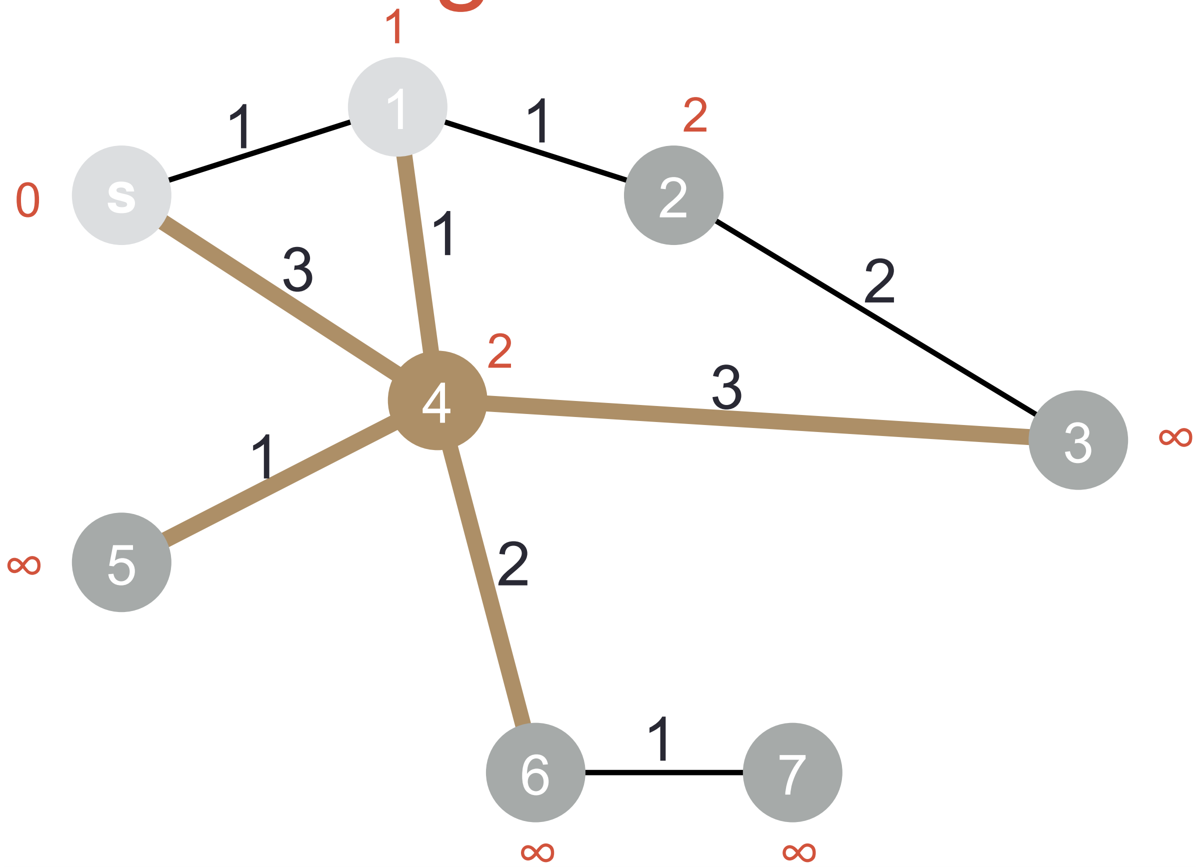


Round 3

Sequential Weighted BFS



$O(D + |E|)$ work where D is the graph diameter

Roun

0 1 2 3 4 5

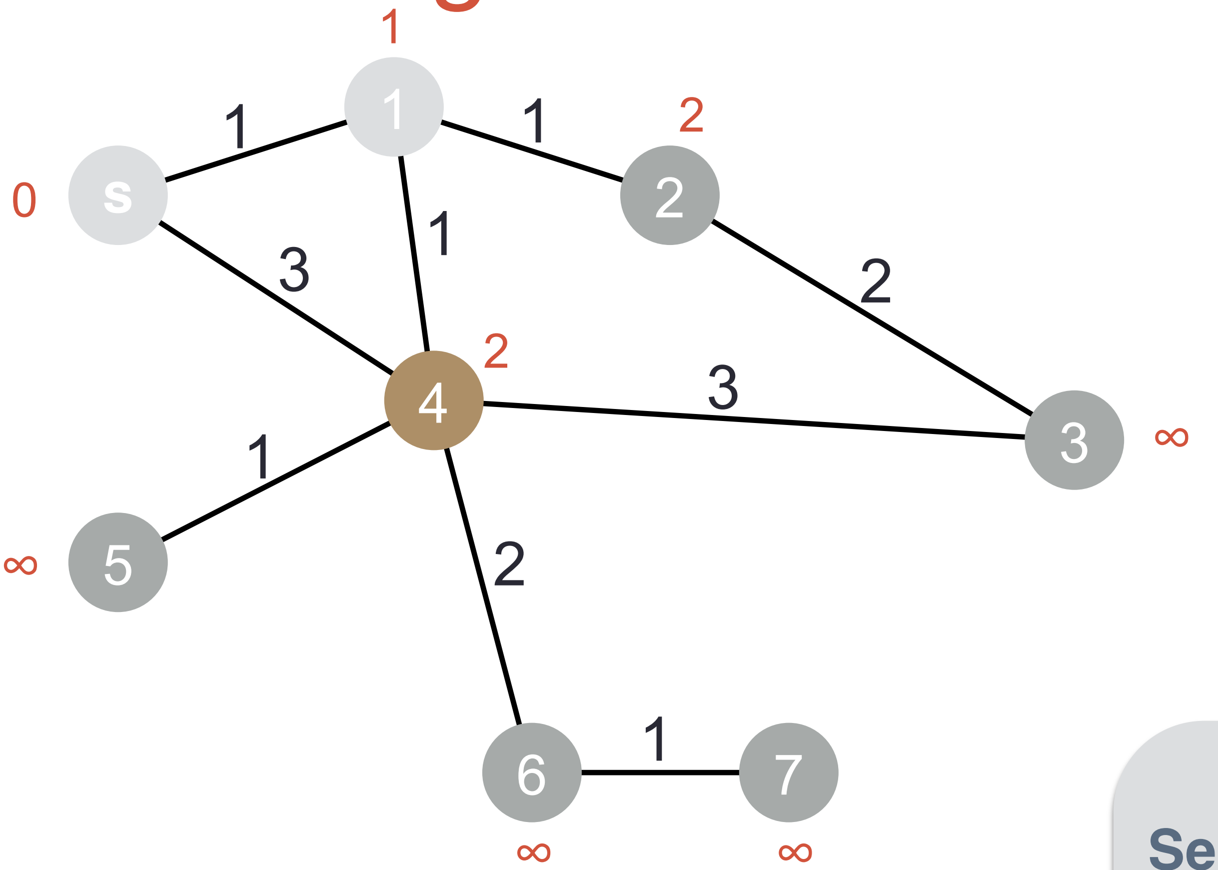
Bucketing

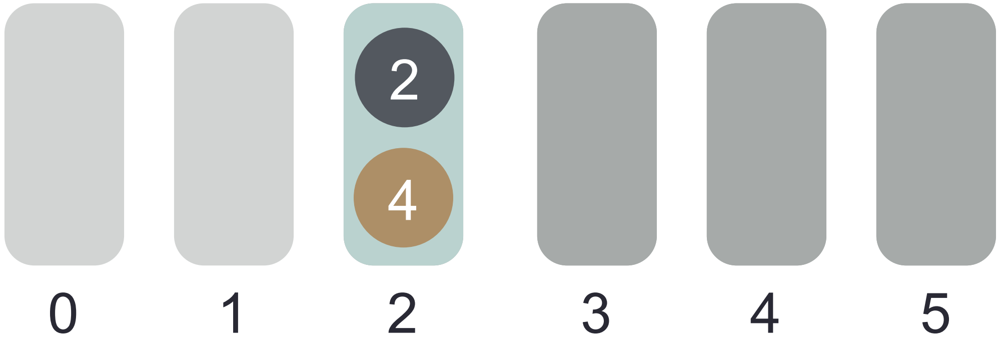
The algorithm uses buckets to *organize work* for future iterations

This algorithm is actually parallelizable

- In each step:
    1. Process all vertices in the next non-empty bucket in parallel
    2. Update buckets of neighbors in parallel

Sequential Weighted BFS

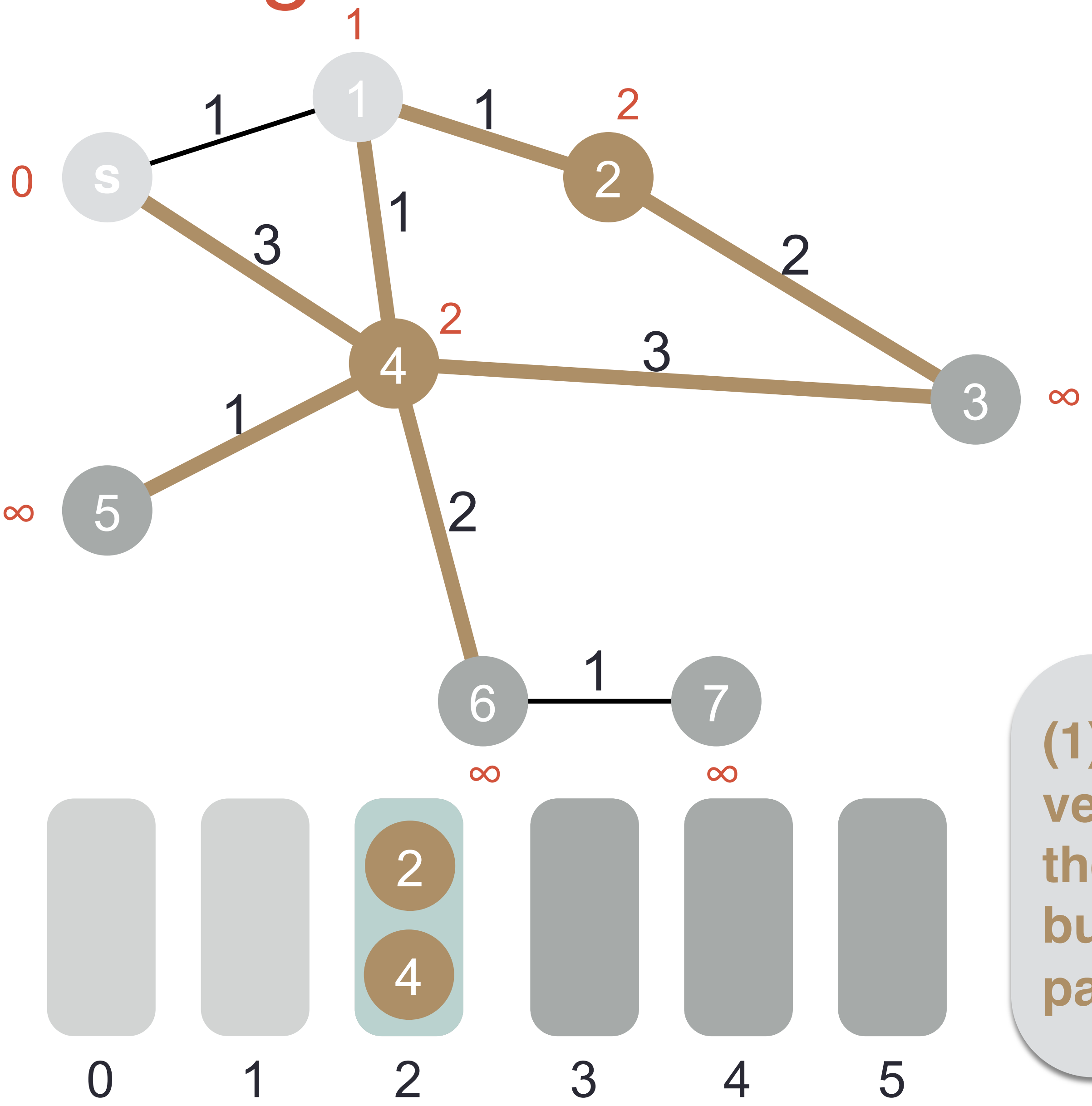




Round 3

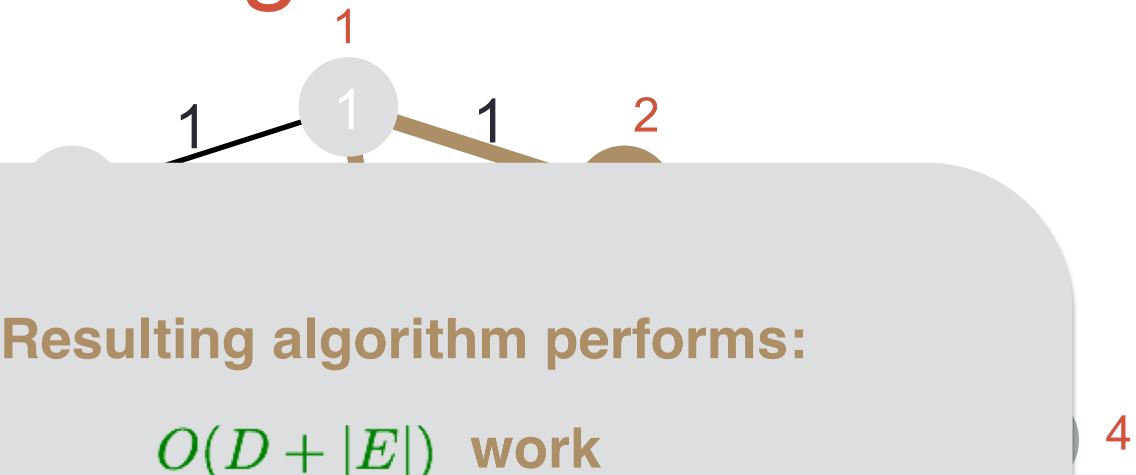
Sequential: process vertices one by one

Parallel Weighted BFS



(1) Process vertices in the same bucket in parallel

Parallel Weighted BFS

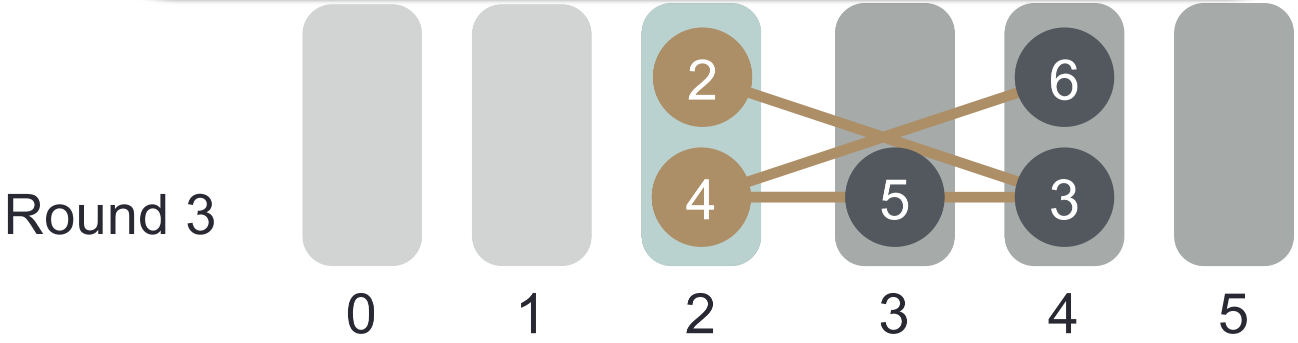


Resulting algorithm performs:

$O(D + |E|)$ **work**

$O(D \log |V|)$ **depth**

(assuming efficient bucketing)



(2) Insert neighbors into buckets in parallel

Parallel Bucketing

Bucketing is useful for more than just weighted BFS

- k-core (coreness)
- Delta Stepping for Single-Source Shortest Paths
- Parallel Approximate Set Cover

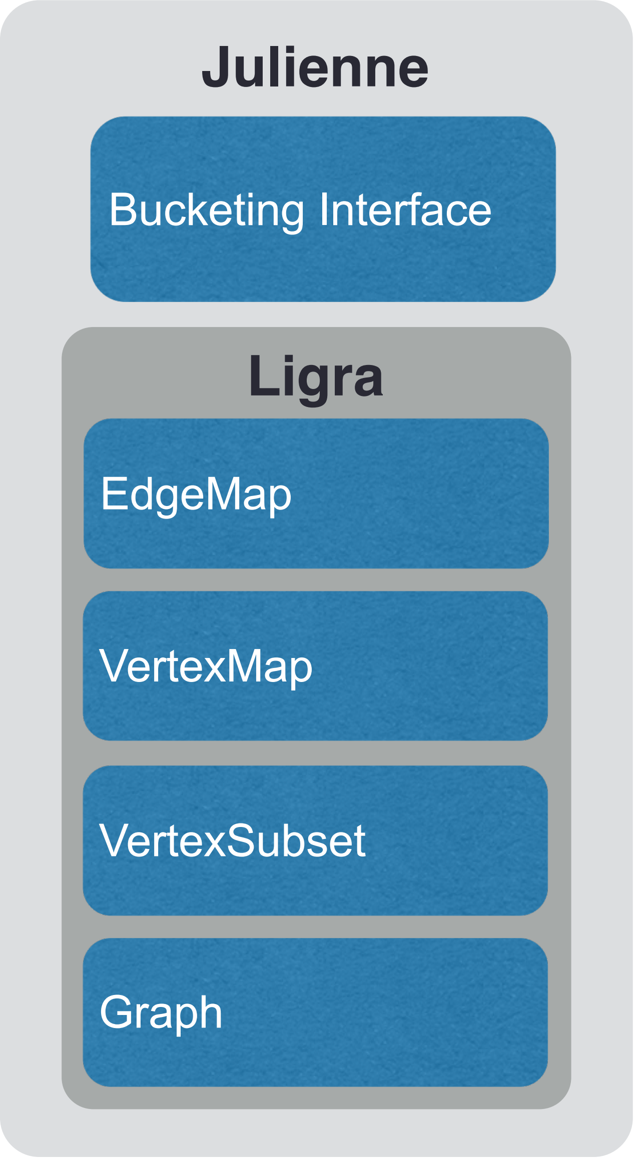
Goals

- Simplify expressing algorithms by using an interface
- Theoretically efficient, reusable implementation

Challenges

1. Multiple vertices insert into the same bucket in parallel
2. Possible to make work-efficient parallel implementations?

Julienne Framework



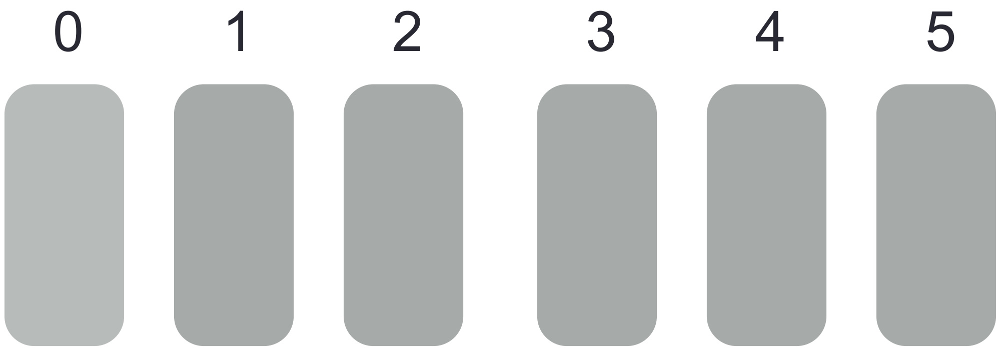
Bucketing Interface:

(1) **MakeBuckets**: Create bucket structure

(2) **NextBucket**: Return the next non-empty bucket (as a VertexSubset)

(3) **UpdateBuckets**: Update buckets of a subset of vertices

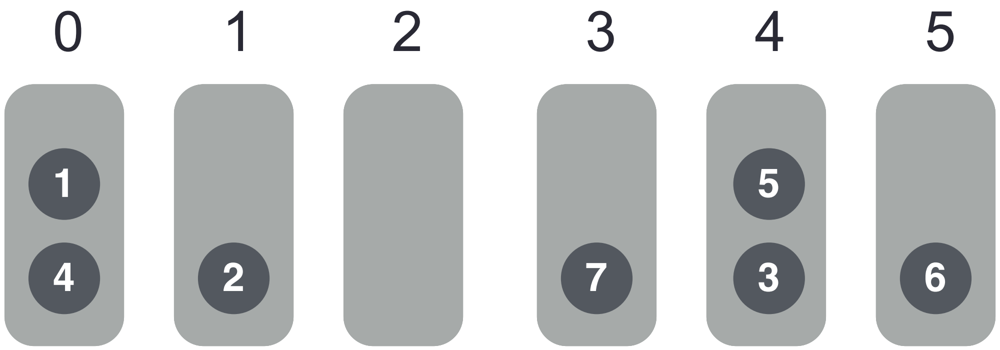
MakeBuckets



$$D(1) = 0, D(2) = 1, D(3) = 4, \ldots$$

Initialize bucket structure

MakeBuckets



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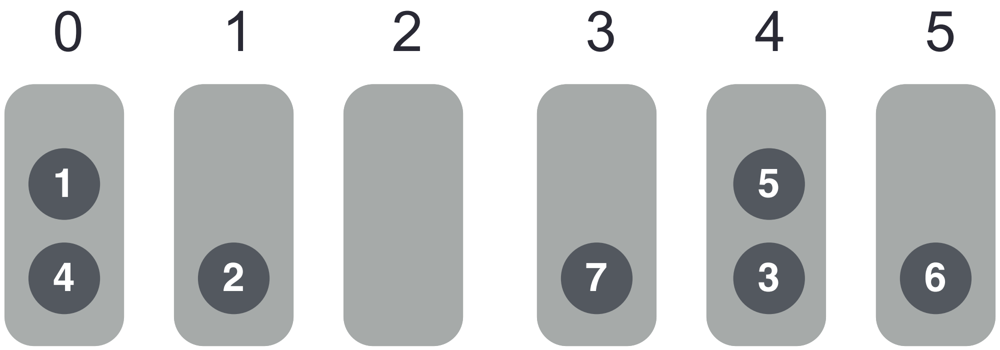
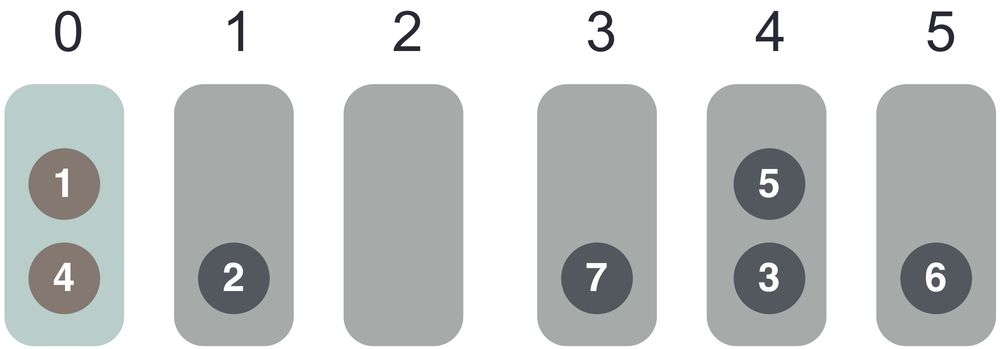
Initialize bucket structure

NextBucket



Extract vertices in the next non-empty bucket

NextBucket



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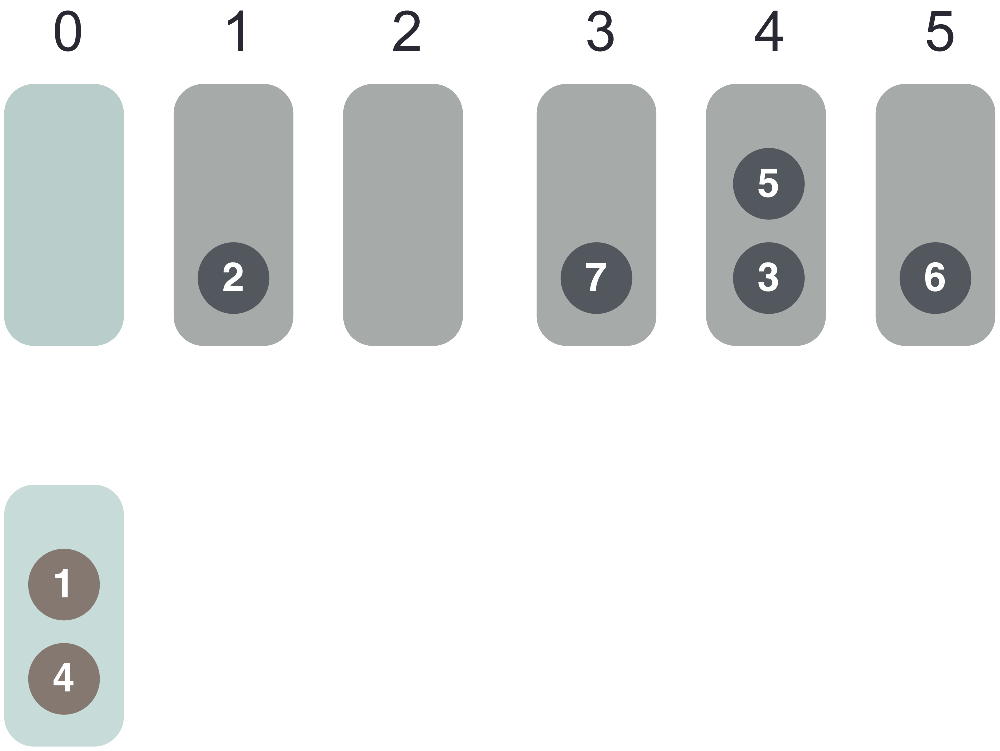
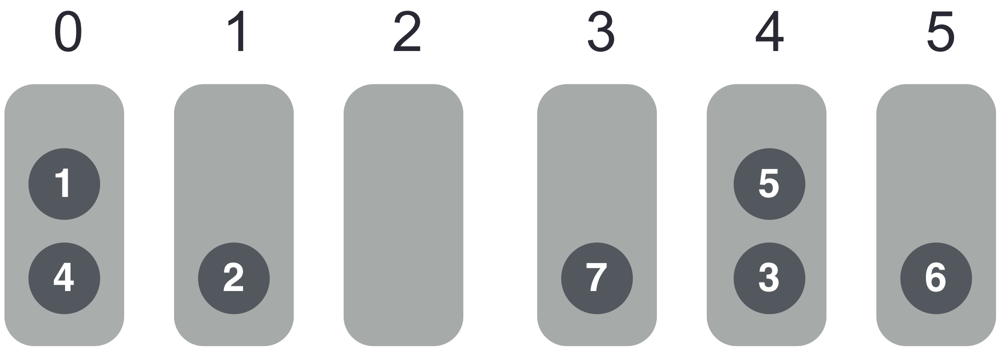


Extract vertices in the next non-empty bucket

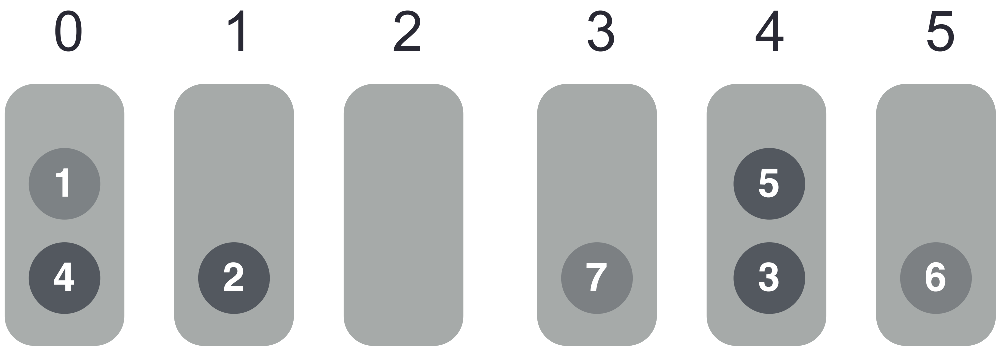
UpdateBuckets



Move vertices to new buckets

Input: array of (vertex, destination bucket) pairs

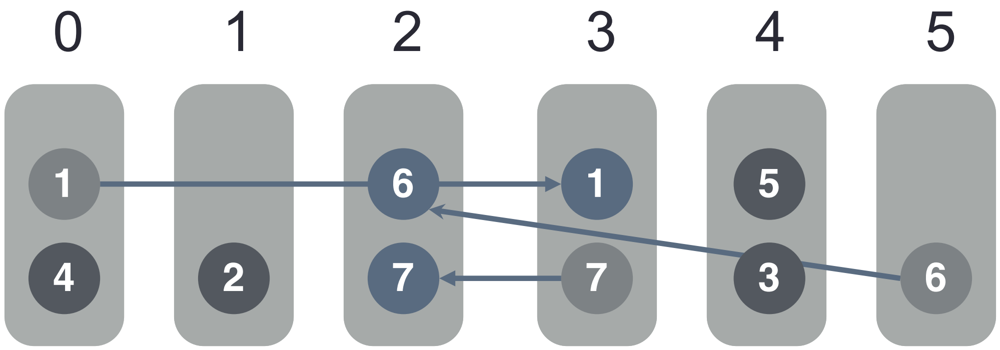
UpdateBuckets



Move vertices to new buckets

[(1,3), (7,2), (6,2)]

UpdateBuckets



Move vertices to new buckets

[(1,3), (7,2), (6,2)]

Sequential Bucketing

Can implement sequential bucketing with:

- n vertices
- T total buckets
- K calls to UpdateBuckets, where each updates the vertices in S_i

  in $O(n + T + \sum_{i=0}^{K} |S_i|)$ work

Implementation:

- Use dynamic arrays
- Update lazily
  - When deleting, leave vertex in bucket
  - When encountering a vertex, check if it has already been processed

Parallel Bucketing

Can implement parallel bucketing with:

- n vertices
- T total buckets
- K calls to UpdateBuckets, where each updates the vertices in S_i
- L calls to NextBucket

in $O(n + T + \sum_{i=0}^{K} |S_i|)$ expected work and

$O((K + L)\log n)$ depth with high probability

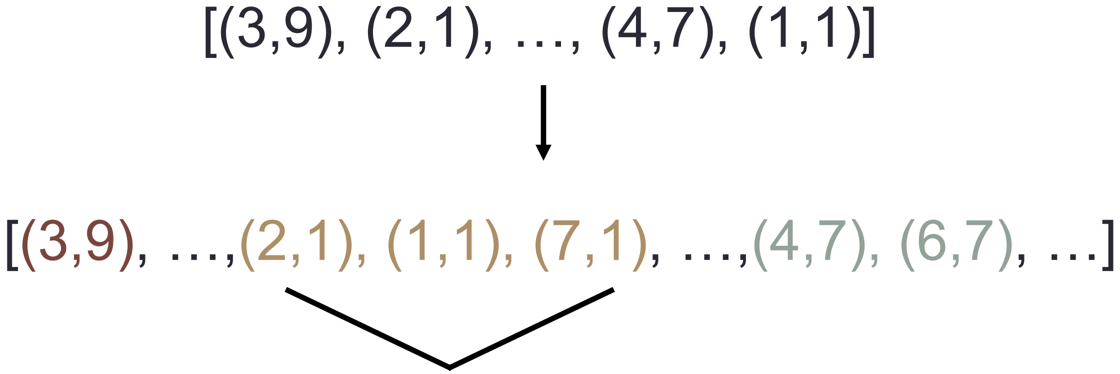
Implementation:

- Use dynamic arrays, delete lazily
- NextBucket: filter out already processed vertices (uses parallel prefix sum, which takes linear work and logarithmic depth)

Parallel Bucketing

UpdateBuckets:

- Use work-efficient semisort [Gu et al. 2015]
- Given *k* (key, value) pairs, semisorts in $O(k)$ expected work and $O(\log k)$ depth with high probability



[(3,9), (2,1), …, (4,7), (1,1)]

[(3,9), …,(2,1), (1,1), (7,1), …,(4,7), (6,7), …]

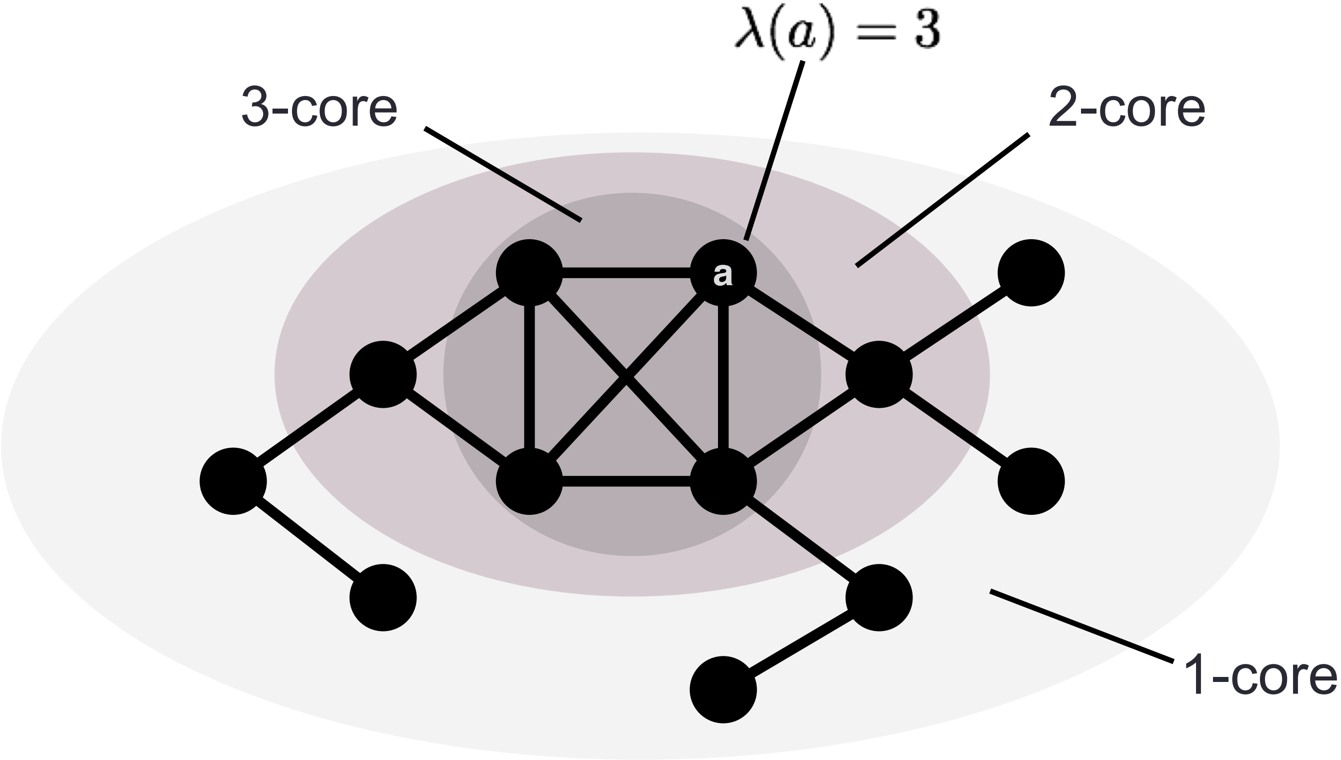
All vertices going to bucket 1

- Compute num. vertices going to each bucket (parallel prefix sum)
- Resize buckets and copy over all vertices in parallel

Example: k-core and Coreness

k-core : maximal connected subgraph of G s.t. all vertices have degree $\geq k$

$\lambda(v)$: largest k-core that v participates in



Can efficiently compute k-cores after computing coreness

Sequential Peeling

Sequential Peeling:

- Bucket sort vertices by degree
- Remove the minimum degree vertex, set its core number
    - Update the buckets of its neighbors

Each vertex and edge is processed exactly once:

$$W = O(|E| + |V|)$$

Parallel Peeling

Existing parallel algorithms:

- Remove all vertices with minimum degree from graph and set their core numbers

Existing parallel algorithms will scan all remaining vertices on each round to find the ones with minimum degree

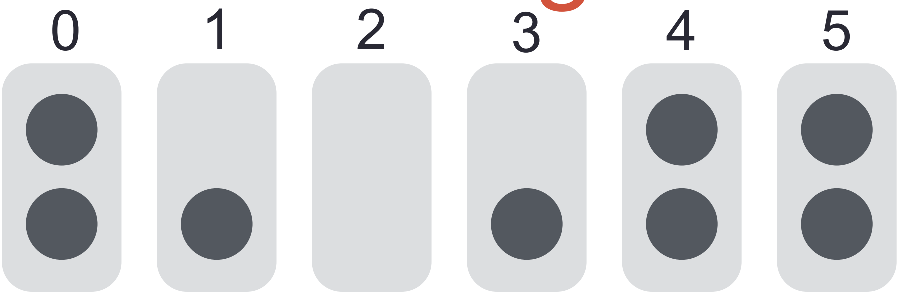
$$W = O(|E| + \rho|V|)$$

$$D = O(\rho \log |V|)$$

$\rho =$ number of peeling steps done by the parallel algorithm

Not work-efficient!

Work-Efficient Peeling

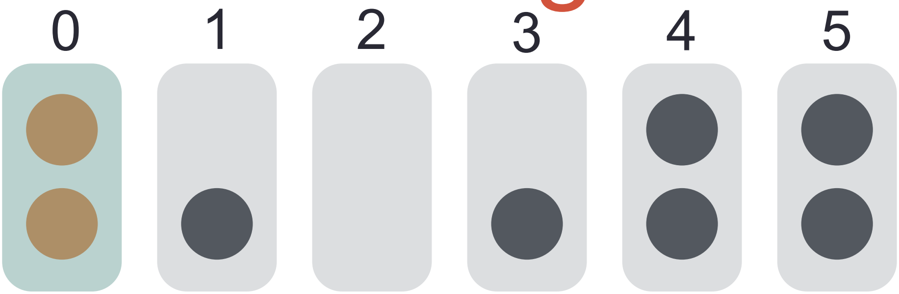


Insert vertices in bucket structure by degree

While not all vertices have been processed yet:

1. Extract the next non-empty bucket, set core numbers

Work-Efficient Peeling



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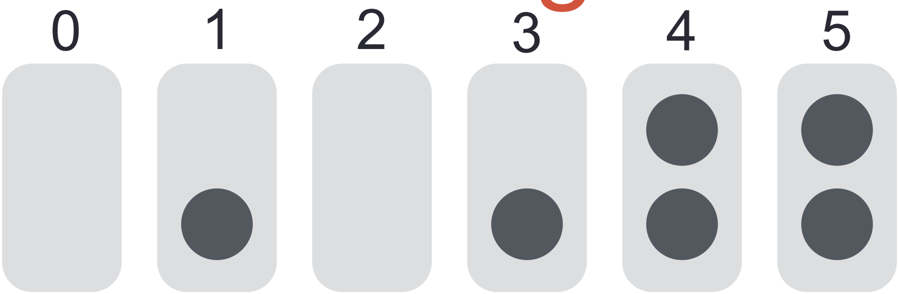

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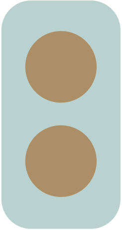
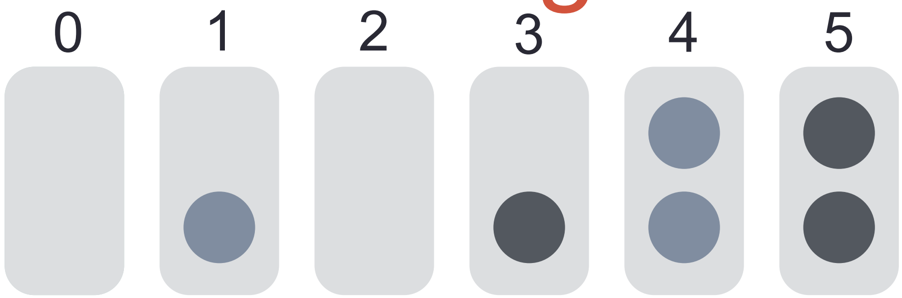


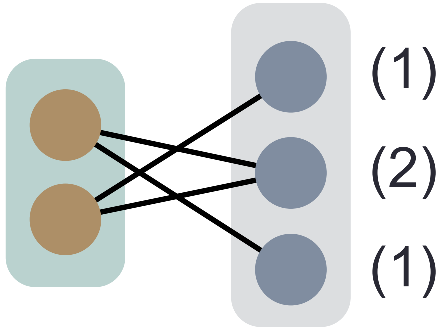
Work-Efficient Peeling



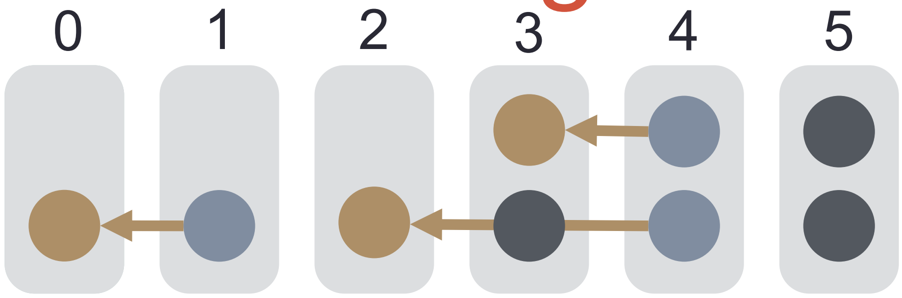
Insert vertices in bucket structure by degree

While not all vertices have been processed yet:

1. Extract the next non-empty bucket, set core numbers
2. Sum edges removed from each neighbor of this frontier



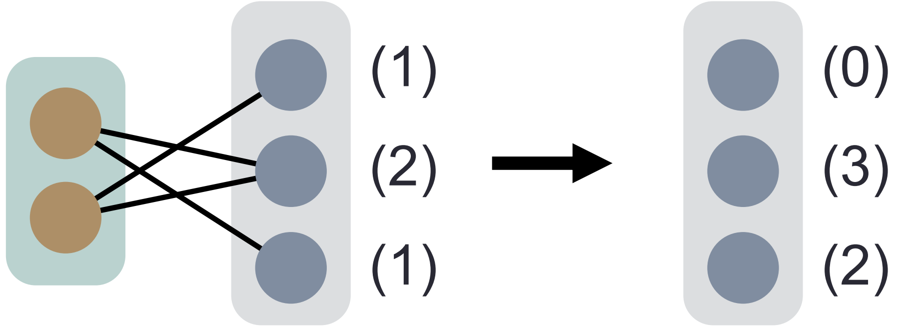
Work-Efficient Peeling



Insert vertices in bucket structure by degree

While not all vertices have been processed yet:

1. Extract the next non-empty bucket, set core numbers
2. Sum edges removed from each neighbor of this frontier
3. Compute the new buckets for the neighbors
4. Update the bucket structure with the (neighbor id, dest bucket)



Work-Efficient Peeling Analysis

We process each edge at most once in each direction:

updates $= O(|E|)$

buckets $\leq$

calls to Ne

calls to Up

Therefore th

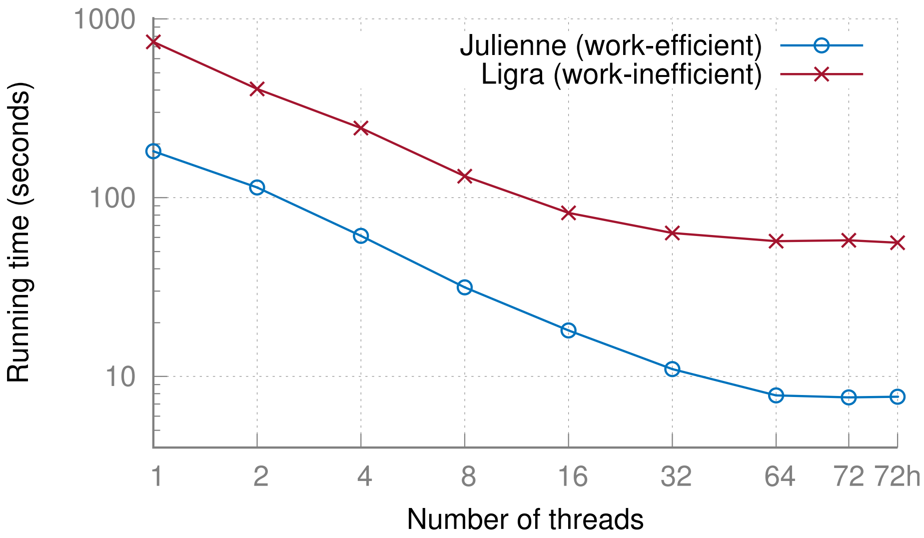
obability

On the Comn K

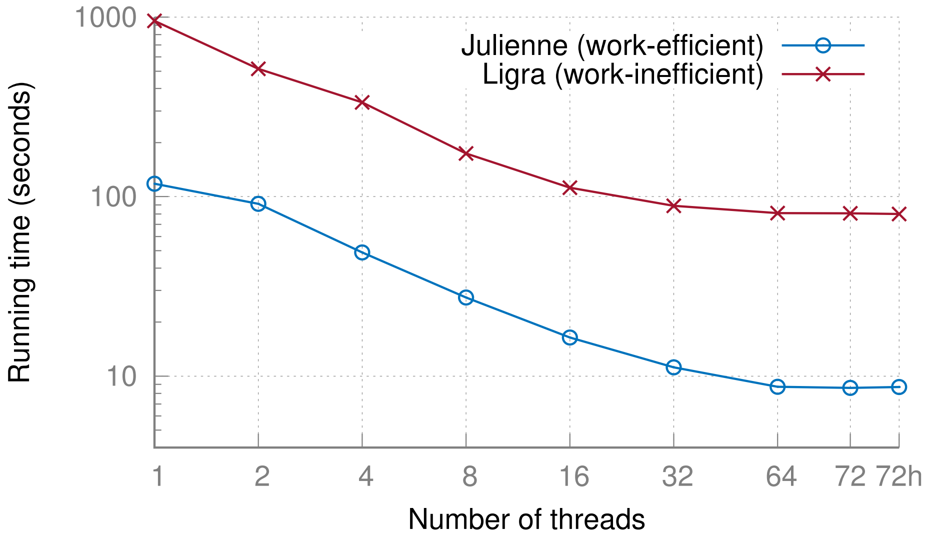
On 72 cores, t the
work-inefficie 3 hours

Efficient peeling using Julienne

Experiments: k-core



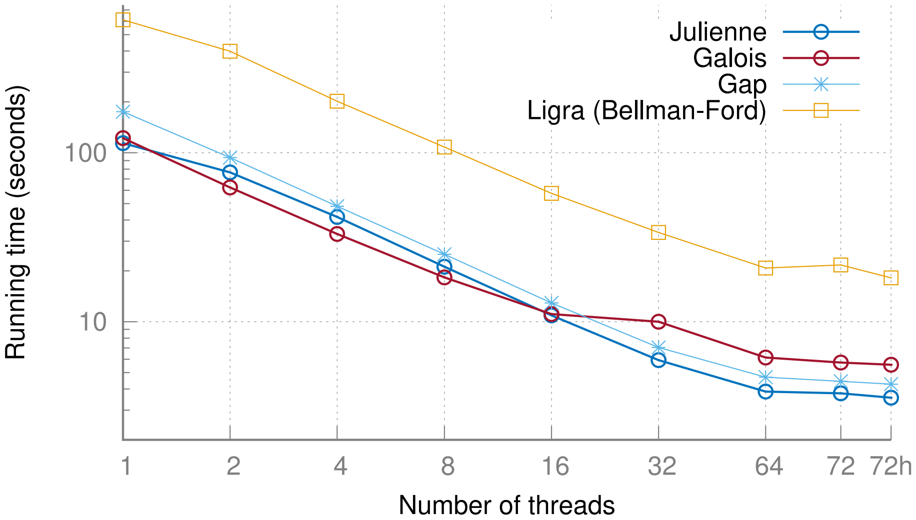
Friendster
(|V| = 121M, |E| = 3.6B, ρ = 10K)



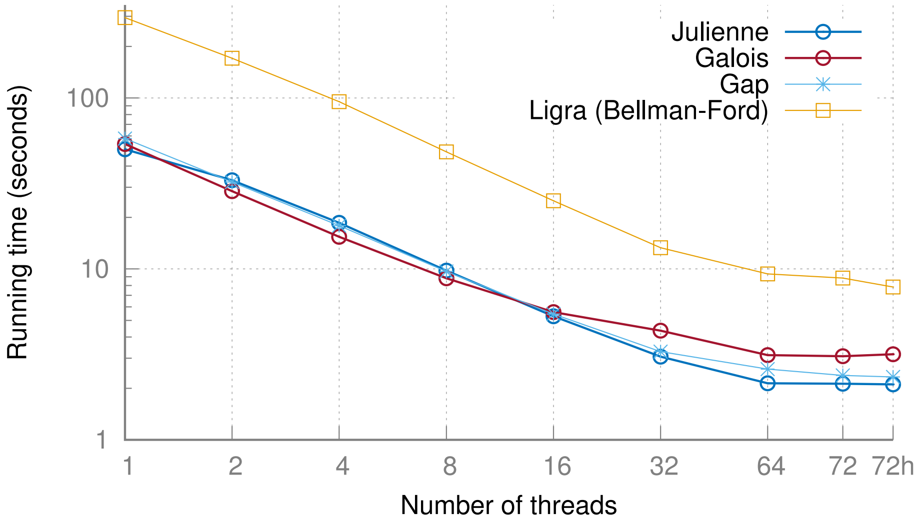
Hyperlink2012-Host
(|V| = 102M, |E| = 3.9B, ρ = 19K)

- 2-9x faster than work-inefficient implementation
- Between 4-41x speedup on 72 cores over sequential peeling
- Speedups are smaller on small graphs with large ρ

Single-Source Shortest Paths



Friendster
(|V| = 121M, |E| = 3.6B, ρ = 10K)



Hyperlink2012-Host
(|V| = 102M, |E| = 3.9B, ρ = 19K)

- 1.8-5.2x faster than (work-inefficient) Bellman-Ford
- Competitive with hand-optimized Single-Source Shortest Paths implementations
- On 72 cores, 18-32x self-relative speedup, 17-30x speedup over DIMACS solver

More Graph Algorithms

- Theoretically-efficient implementations of over a dozen other graph algorithms
- Compression was crucial in running on 1TB machine
  - Compressed edge lists using delta encoding and variable-length codes
- Theoretically-efficient parallel primitives on compressed edge lists
  - Map, Map-Reduce, Filter, Pack, Intersect

Scaling to Largest Graph

	Asymmetric	Symmetrized
Common Crawl	3.5 billion vertices 128 billion edges (540 GB)	3.5 billion vertices 225 billion edges (928 GB)

- 72-core machine with 1TB RAM

Algorithm	Time
k-core	193 sec
Weighted BFS	58 sec
Biconnected components	201 sec
Strongly connected components	182 sec
Minimum spanning forest	228 sec
Maximal independent set	34 sec
Maximal matching	126 sec
Triangle counting	1470 sec

Algorithm	Time
Breadth-first search	12 sec
Connected components	38 sec
Bellman-Ford	53 sec
Betweenness centrality (1 source)	40 sec
Low-diameter decomposition	18 sec
Graph coloring	174 sec
Approximate set cover	104 sec
PageRank (1 iteration)	28 sec

- Outperforms reported numbers for this graph
- For many algorithms, no published results for this graph

Conclusion

- Theoretically-efficient parallel algorithms can be fast and scalable
- Can process largest graphs on a single multicore server with 1TB of RAM

- Julienne framework available at https://github.com/jshun/ligra
- All of our theoretically-efficient graph algorithms are available at https://github.com/ldhulipala/gbbs

Final Project

- Project presentations on Thursday
  - 5 minutes per team member, and 5 minutes for Q&A
  - Problem and motivation
  - Prior work
  - Your technical contributions
  - Challenges encountered
  - Experimental results
  - Work breakdown among team members
- Project report due on Thursday

- Comm Lab available to improve your presentation and write-up

Course Summary

- Congratulations on making it through all the lectures!
- Opportunities to continue doing research
  - UROP (http://uaap.mit.edu/research-exploration/urop)
  - SuperUROP (https://superurop.mit.edu/)
  - M.Eng.
  - Ph.D.
- Look out for relevant seminars (seminars@csail.mit.edu)
- Conferences relevant to algorithm engineering: SPAA, ALENEX, ESA, SEA, PODC, IPDPS, SC, PPoPP, and more