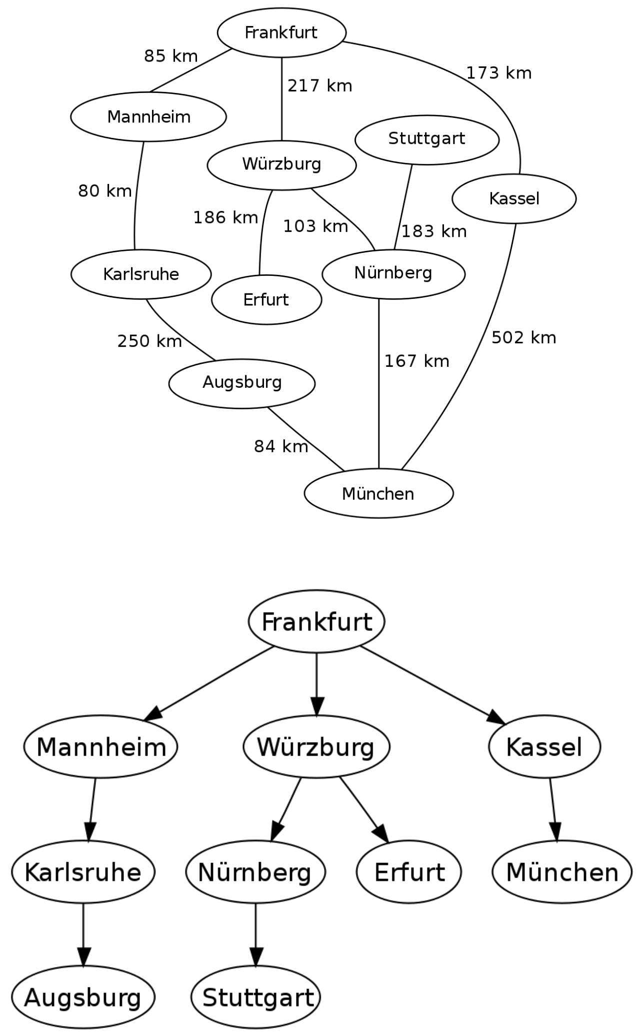
Direction-Optimizing Breadth-First Search

Scott Beamer, Krste Asanovic, David Patterson

Presented by Siddhartha Jayanti

Problem

- Speed-Up Parallel BFS Computation
  - Mark Visited Vertices
  - Compute BFS TREE



Breadth First Search Algorithm

```
function breadth-first-search(vertices, source)
    frontier ← {source}
    next ← {}
    parents ← [-1,-1,...-1]
    while frontier ≠ {} do
        top-down-step(vertices, frontier, next, parents)
        frontier ← next
        next ← {}
    end while
    return tree
```

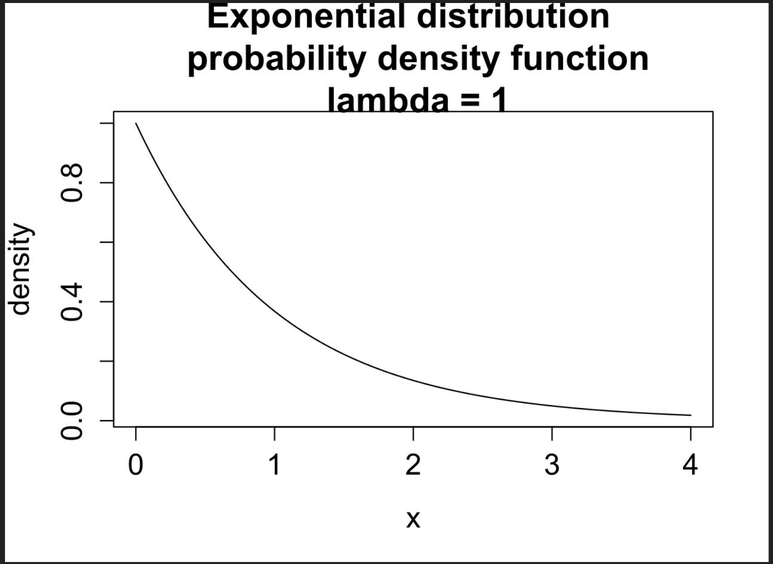
Fig. 1. Conventional BFS Algorithm

```
function top-down-step(vertices, frontier, next, parents)
    for v ∈ frontier do
        for n ∈ neighbors[v] do
            if parents[n] = -1 then
                parents[n] ← v
                next ← next ∪ {n}
            end if
        end for
    end for
```

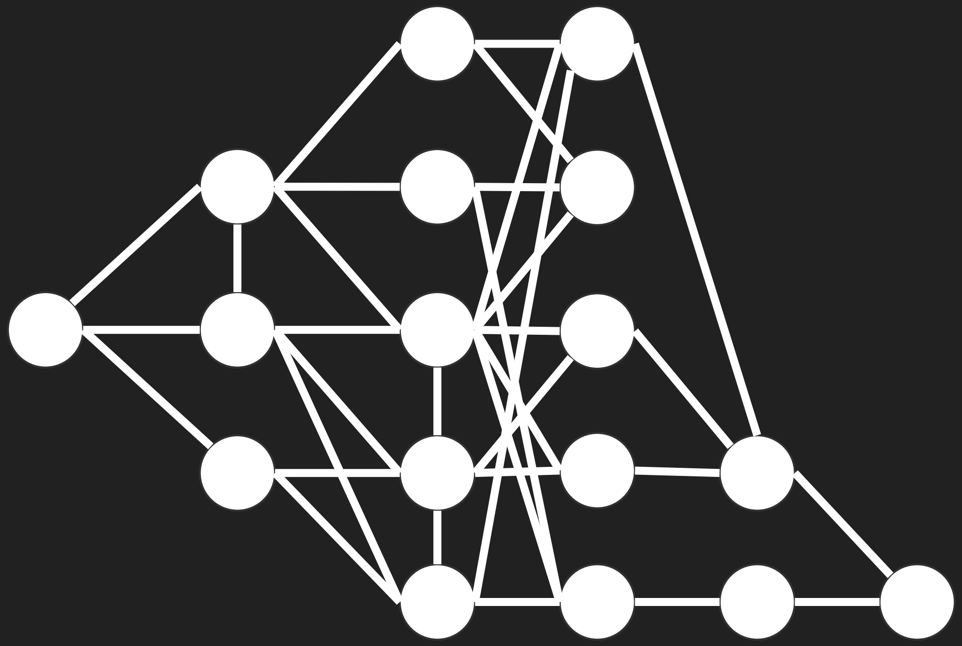
Fig. 2. Single Step of Top-Down Approach

Technique

- Reduce EDGE Traversals
  - Worst-Case: O(*n* + *m*)
  - Best-Case: O(*n*)
- Can’t always do better
  - e.g. Path
  - How about common practical case?
- Motivation
  - Social Network
  - Exponential Law for degrees



Main Concept #1: Frontier



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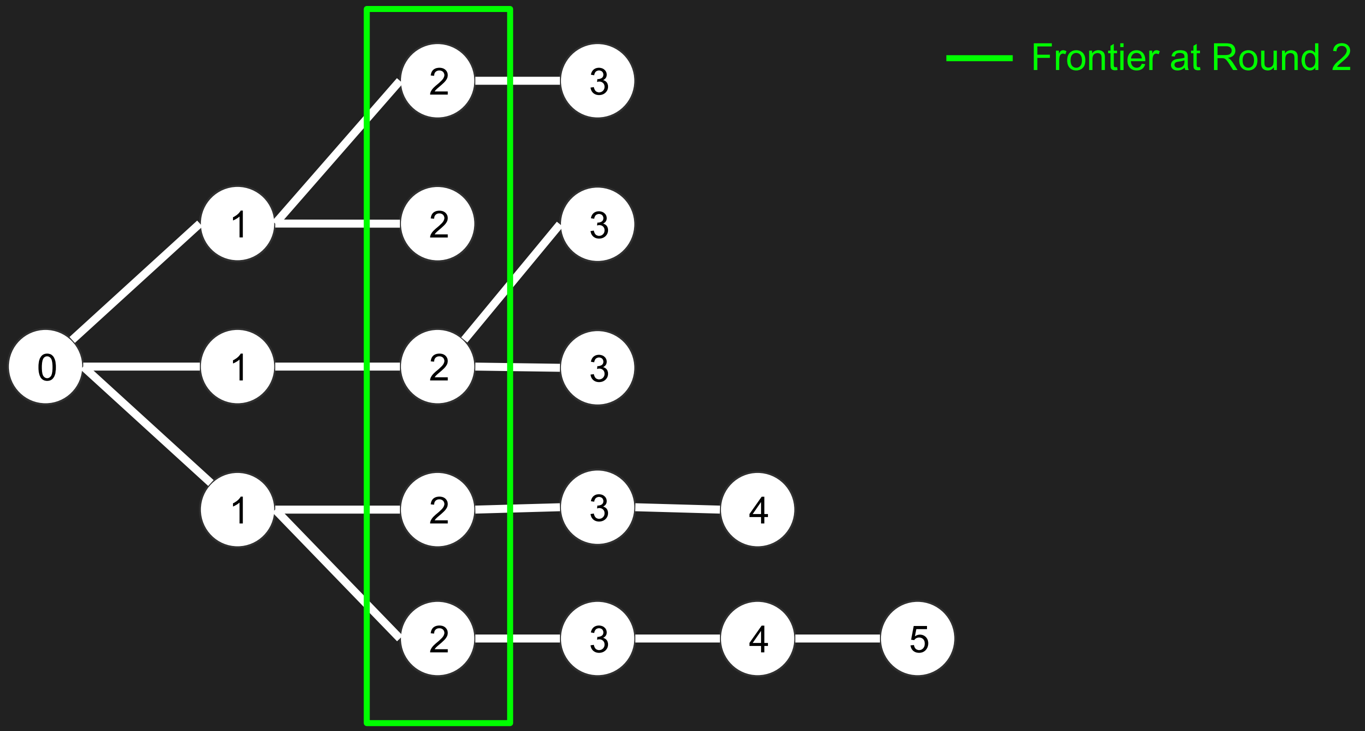
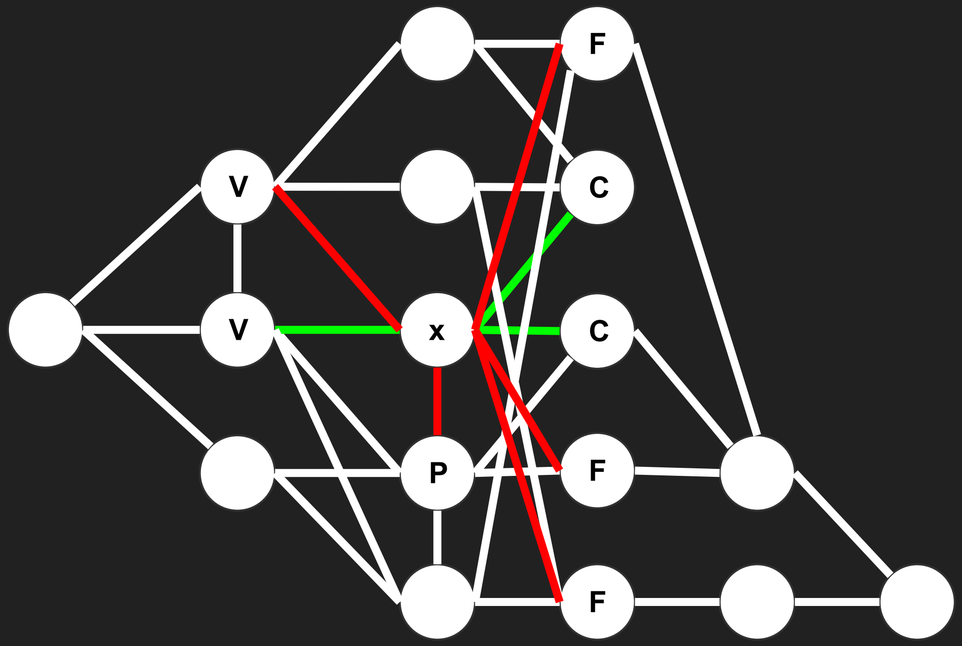
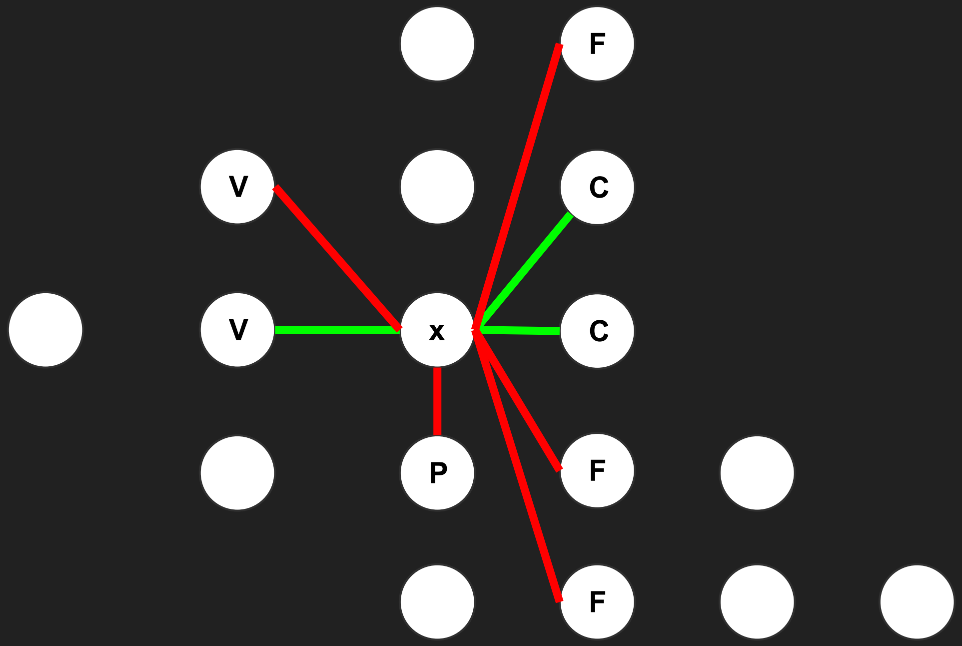


Main Concept #2: Classification of Nodes



V	valid parent
P	peer
F	failed child
C	claimed child

Main Concept #2: Classification of Nodes



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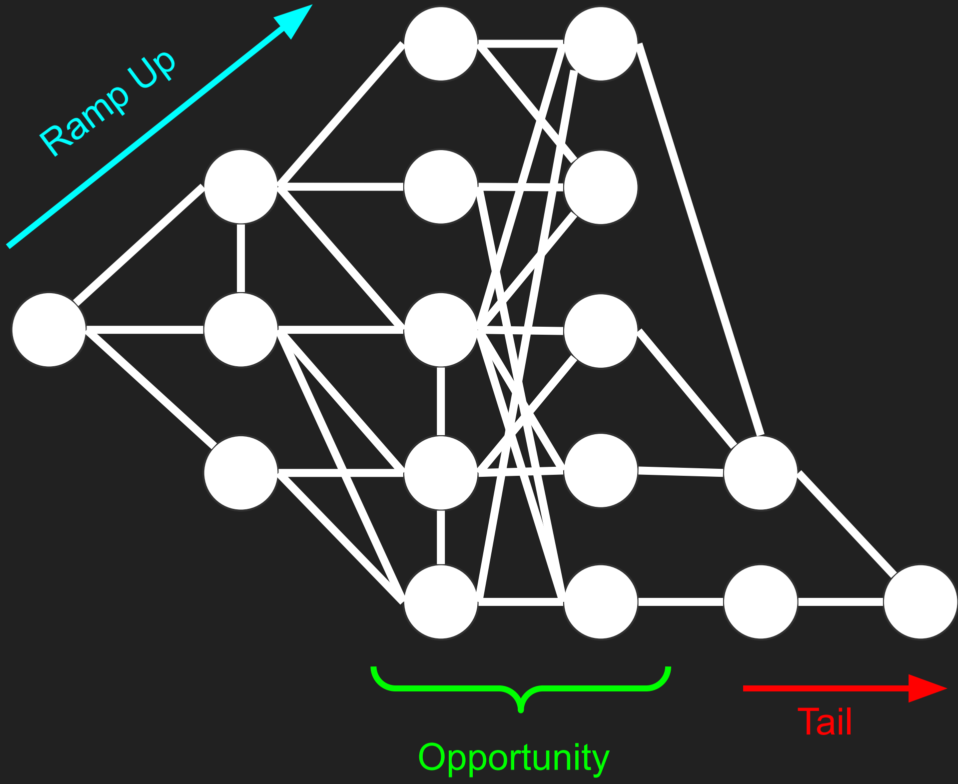
Main Concept #3: Social Network Structure



Empirical Justification

In Opportunity Zone:

- Few Claimed Children
- Lots of Failed Children

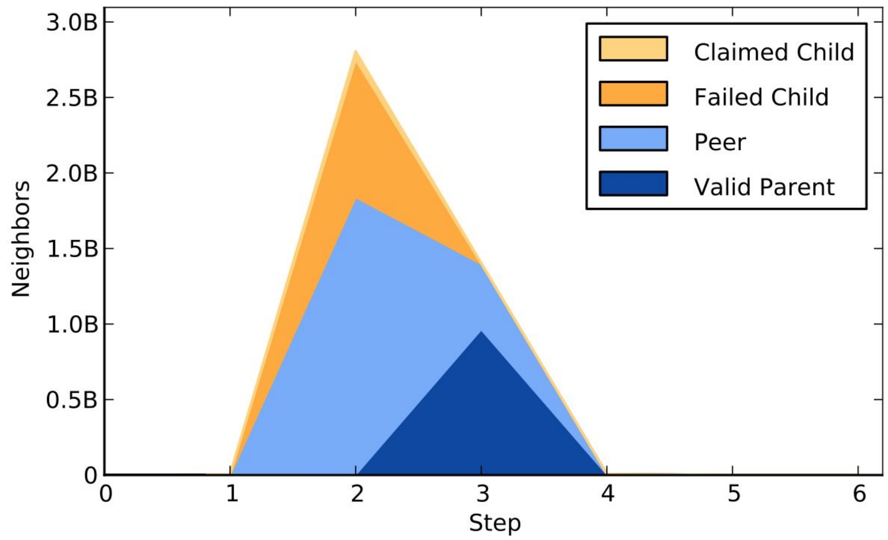


Fig. 3. Breakdown of edges in the frontier for a sample search on kron27 (Kronecker generated 128M vertices with 2B undirected edges) on the 16-core system.

Empirical Justification

In Ramp Up:

- Lots of Claimed Children

In Opportunity Zone:

- Lots of Failed Children

In Tail Zone:

- Most parents are Valid

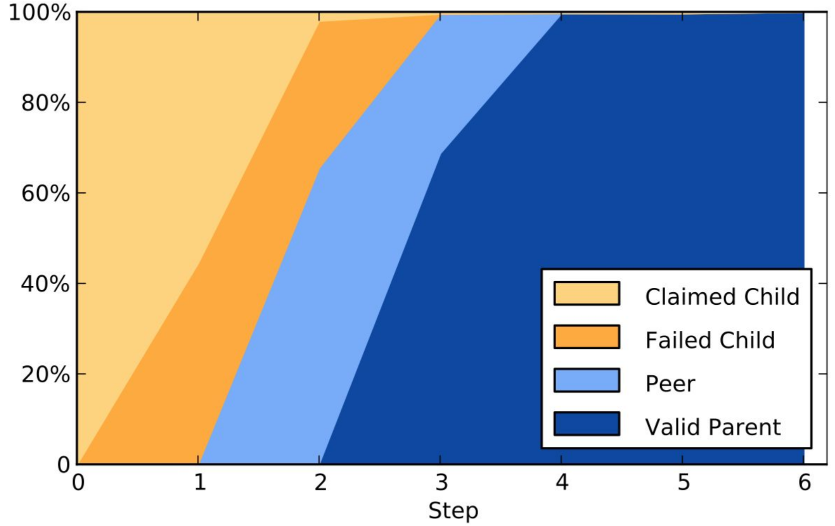


Fig. 4. Breakdown of edges in the frontier for a sample search on `kron27` (Kronecker generated 128M vertices with 2B undirected edges) on the 16-core system.

Breadth First Search: Bottom-Up Step

```
function breadth-first-search(vertices, source)
    frontier ← {source}
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```
function top-down-step(vertices, frontier, next, parents)
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        for n ∈ neighbors[v] do
            if parents[n] = -1 then
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                next ← next ∪ {n}
            end if
        end for
    end for
```

Fig. 2. Single Step of Top-Down Approach

Breadth First Search: Bottom-Up Step

```
function bottom-up-step(vertices, frontier, next, parents)
    for v ∈ vertices do
        if parents[v] = -1 then
            for n ∈ neighbors[v] do
                if n ∈ frontier then
                    parents[v] ← n
                    next ← next ∪ {v}
                    break
                end if
            end for
        end if
    end for
```

Fig. 5. Single Step of Bottom-Up Approach

Comparison

Advantages of Top-Down:

- Frontier is small & lots of neighbors

Advantages of Bottom-Up:

- Frontier large compared to remaining vertices
- Can stop search early
- No write-contention

Heuristic

n_f = # vertices in frontier

m_f = # edges to check from the frontier

m_u = # edges to check from unexplored vertices

α, **β** - tuning parameters

$$m_f > \frac{m_u}{\alpha} = C_{TB}$$

$$n_f < \frac{n}{\beta} = C_{BT}$$

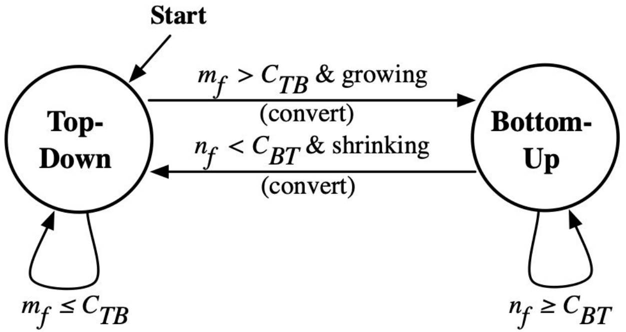


Fig. 7. Control algorithm for hybrid algorithm. (convert) indicates the frontier must be converted from a queue to a bitmap or vice versa between the steps. Growing and shrinking refer to the frontier size, and although they are typically redundant, their inclusion yields a speedup of about 10%.

Hybrid-heuristic is robust to tuning α

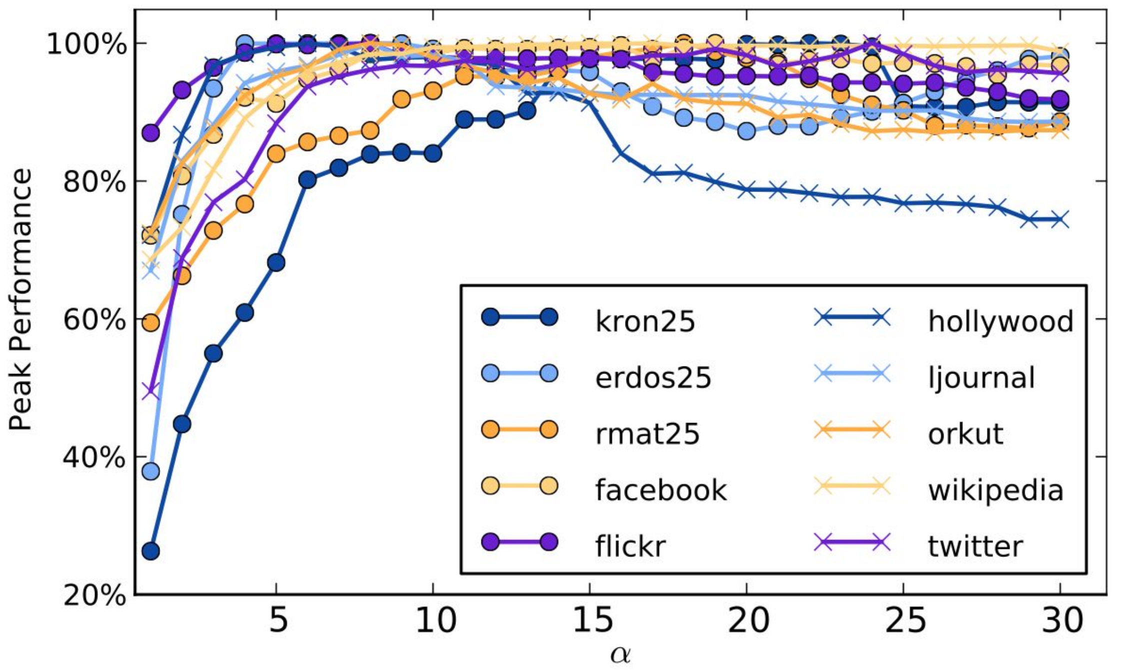


Fig. 8. Performance of *hybrid-heuristic* on each graph relative to its best on that graph for the range of α examined.

Hybrid-heuristic is robust to tuning **β**

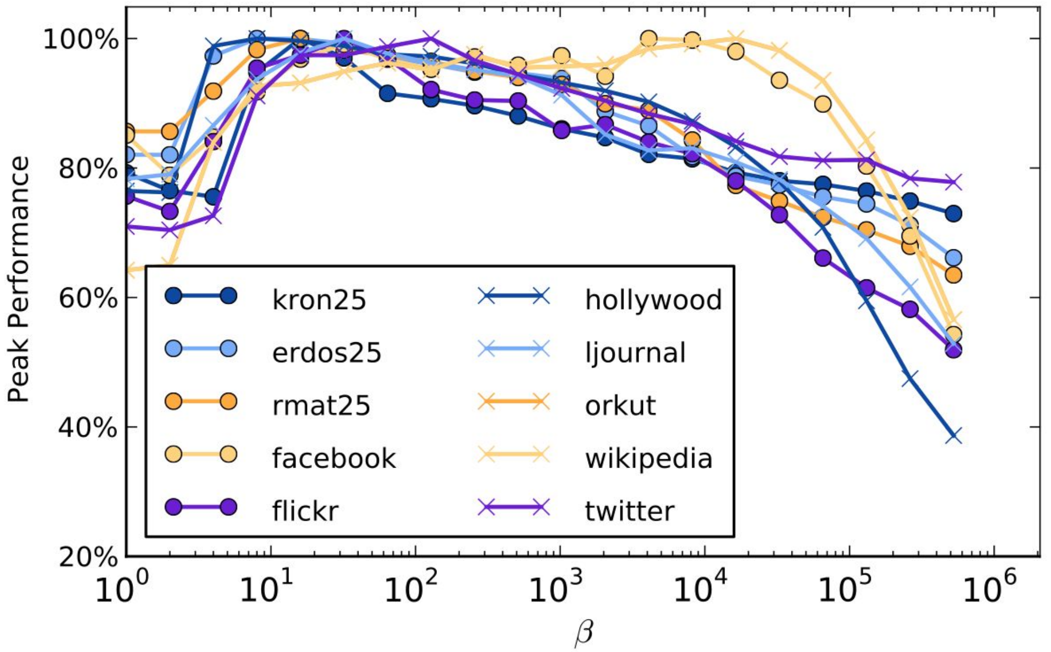


Fig. 9. Performance of *hybrid-heuristic* on each graph relative to its best on that graph for the range of β examined.

Performance of Method (dark purple vs. light blue)

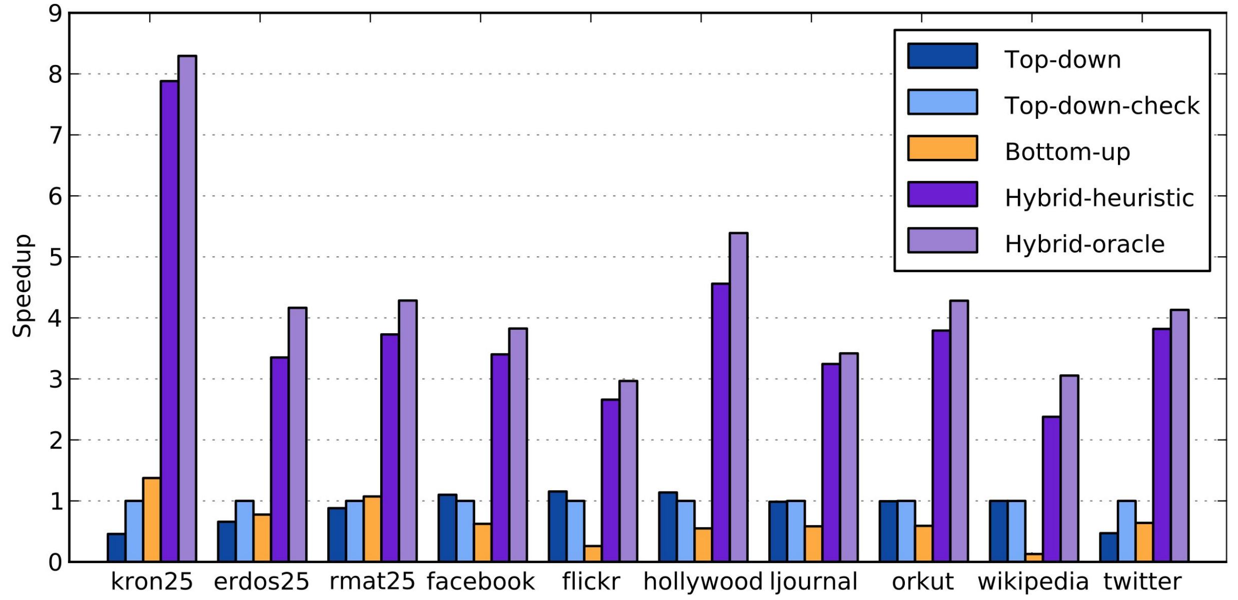
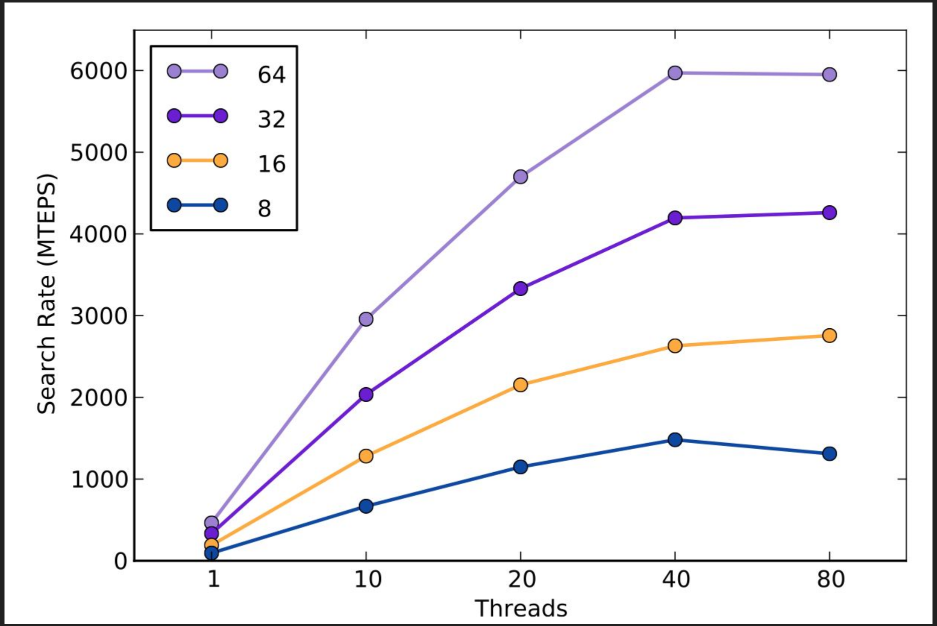


Fig. 10. Speedups on the 16-core machine relative to *Top-down-check*.

Hybrid is 2 to 8 times as fast as original top-down-check algorithm

Additional Threads Help Speed Up - Hyperthreading Doesn't



Conclusion

- Works well if Low Effective Diameter
- Top Down -> Bottom Up -> Top Down
- High-diameter graphs don't benefit from bottom-up, but are easier to parallelize

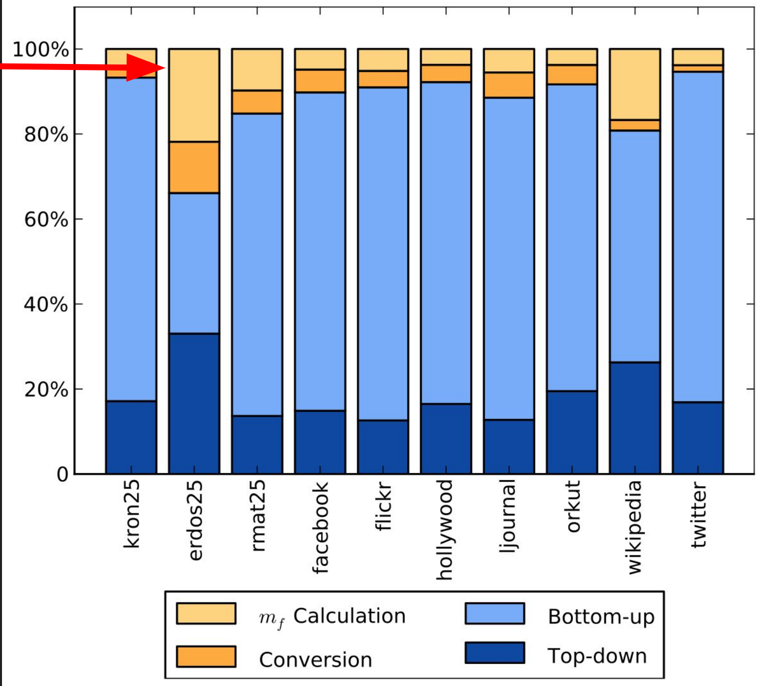


Fig. 12. Breakdown of time spent per search.

Questions

- How important is parallelism for this idea?
- Are there other graph problems that could benefit from this type of thought?
  - Could parallel Dijkstra use this idea?
  - Something else?
- What are experiments that you'd like to see that were missing?
- When might normal BFS be better than the hybrid algorithm?

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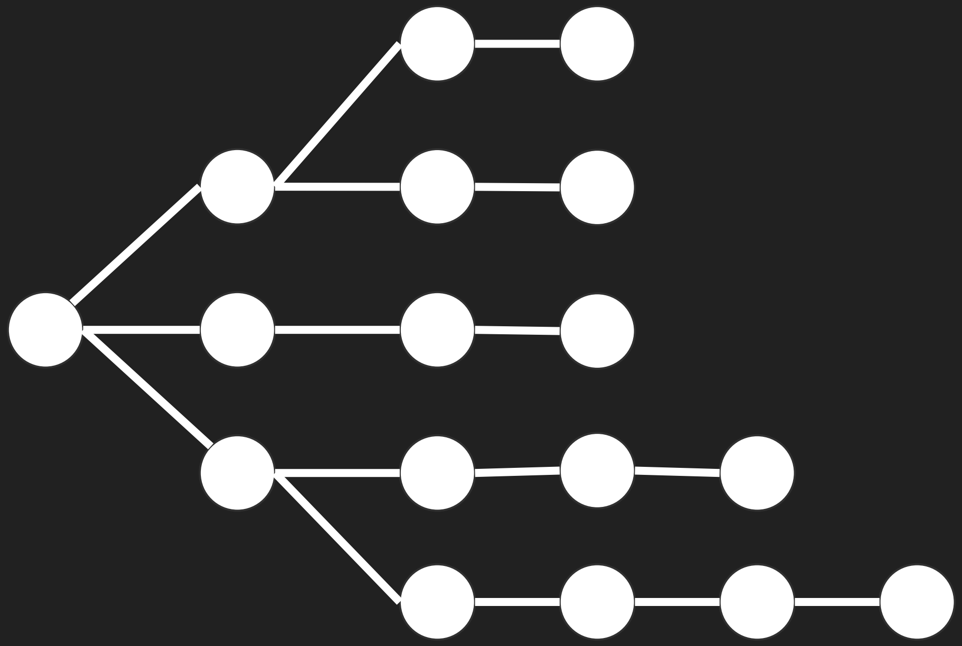

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