

Exploring the Design Space of Static and Incremental Graph Connectivity Algorithms on GPUs

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ABSTRACT

Connected components and spanning forest are fundamental graph algorithms due to their use in many important applications, such as graph clustering and image segmentation. GPUs are an ideal platform for graph algorithms due to their high peak performance and memory bandwidth. While there exist several GPU connectivity algorithms in the literature, many design choices have not yet been explored. In this paper, we explore various design choices in GPU connectivity algorithms, including sampling, linking, and tree compression, for both the static as well as the incremental setting. Our various design choices lead to over 300 new GPU implementations of connectivity, many of which outperform state-of-the-art. We present an experimental evaluation, and show that we achieve an average speedup of 2.47x speedup over existing static algorithms. In the incremental setting, we achieve a throughput of up to 48.23 billion edges per second. Compared to state-of-the-art CPU implementations on a 72-core machine, we achieve a speedup of 8.26–14.51x for static connectivity and 1.85–13.36x for incremental connectivity using a Tesla V100 GPU.

CCS CONCEPTS

• Theory of computation → Parallel algorithms.

KEYWORDS

Connected components, Graph algorithms, GPU algorithms, Spanning forest

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1 INTRODUCTION

Connected components (connectivity) is a fundamental graph problem that plays a critical role in many graph applications. Given an undirected graph with n vertices and m edges, the problem assigns each vertex a label such that vertices that are reachable from each

other have the same label, and otherwise have different labels [25]. Connectivity algorithms are used in many applications, such as computer vision [31, 43], VLSI design [61], and social analysis [41]. Graph connectivity is also a key subroutine to solve other graph algorithms, such as biconnectivity [86] and clustering [33, 72, 90, 91], and some of these algorithms require many calls to graph connectivity. As such, there has been a large amount of work on efficient parallel connectivity algorithms [2–7, 11, 12, 14, 15, 17, 20–23, 27, 34, 36–40, 42, 46–48, 50, 55–59, 62, 63, 67, 68, 73–75, 77–79, 81–83, 85, 87].

Graphics processing units (GPUs) are attractive devices for performing graph computations because of their high computing power and memory bandwidth. However, achieving high performance using GPUs is challenging due to several factors, including uncoalesced memory access, insufficient parallelism to tolerate high memory latency, load imbalance, and thread divergence. Several GPU connectivity implementations have been proposed in the literature [13, 24, 49, 65, 71, 85, 88], but we found that there are many algorithmic choices and optimizations that have not been thoroughly explored for GPU connectivity. The goal of this paper is to explore this large design space to better understand how different choices affect performance.

In this paper, we study *min-based connectivity algorithms* [52, 53, 62, 73, 78, 83], which are based on vertices propagating labels to other connected vertices, and keeping the minimum label received. At convergence, vertices will have the same label if and only if they are in the same connected component. A large number of our algorithms are based on using a union-find data structure for maintaining disjoint sets. The data structure maintains a tree for each sub-component found so far, and joins trees to merge sub-components that are connected. We also study several other min-based algorithms that maintain trees, but not using a disjoint set data structure. All existing GPU connectivity implementations are min-based; however, there are various design choices that have not been explored, such as different rules for searching and compressing union-find trees and for propagating labels. Furthermore, these algorithms can be improved by using sampling as a preprocessing step to remove vertices in a large component from consideration, so that the remaining steps are more efficient. Sutton et al. [85] provide one instantiation of sampling combined with a particular union-find algorithm. Inspired by their work, we explore different sampling strategies in this paper, and combine them with each of our algorithms to sweep the search space.

To implement different connectivity algorithms, we designed the GConn framework, which is an extension of the ConnectIt framework [29] for multicore CPUs. However, achieving high performance for connectivity algorithms on GPUs requires significant effort beyond what is provided in ConnectIt for two reasons. The first reason is that the programming models on CPUs and GPUs are

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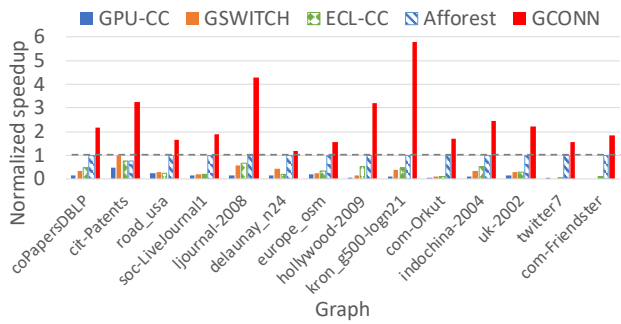


Figure 1: Normalized speedup of four existing implementations/libraries and GConn. The fastest existing implementation’s performance is normalized to 1.

different, which required us to significantly rewrite the codebase. The second reason is that the bottlenecks on CPUs and GPUs are different (e.g., on GPUs, performance can be easily degraded from uncoalesced memory accesses, low parallelism, and heavy use of atomics), and this required us to apply GPU-specific optimizations to achieve high performance. GConn contains several GPU-specific optimizations: edge reorganization, which is specific to our connectivity algorithms, and CSR coalescing and vertex gathering, which are commonly used in other graph algorithms.

In this paper, we generate a total of 339 different connectivity implementations and evaluate their performance. With our comprehensive study, we are able to obtain the fastest GPU connectivity algorithms to date. Figure 1 shows the normalized performance of the fastest implementation in GConn compared to four state-of-the-art GPU implementations: GPU-CC [82], GSWITCH [65], ECL-CC [49], and Afforest [85]. We achieve an average speedup of 2.48x over the fastest implementation for each graph input. Although we consider many connectivity implementations, we note that based on the results of our experimental study, practitioners only need to consider a handful of these implementations to obtain high performance.

In addition to connected components, most of our algorithms solve the related problem of computing a spanning forest of a graph. Furthermore, we extend our algorithms to the incremental setting, where the connected components or spanning forest is updated upon new edge arrivals. Our incremental algorithms are able to achieve throughputs of up to 48.23 billion edges per second, which improves upon state-of-the-art for GPUs—EvoGraph [76]—by orders of magnitude based on a rough comparison of reported numbers since their code is not available. Additionally, we compare our GPU implementations to the CPU implementations in ConnectIt on a 72-core CPU and show that we achieve speedups of 8.27–17.48x for static connectivity, and 1.85–13.36x for incremental connectivity on a Tesla V100 GPU that costs about half as much as the CPU. Finally, we perform an analysis of different design choices to explain where the performance benefits of our fastest implementations are coming from. GConn is publicly available at <https://github.com/hochawa/gconn>.

Summary of Contributions. We believe that this paper presents the most comprehensive study of GPU implementations of connectivity for both static and incremental connectivity to date. By performing this extensive study, we provide an understanding of

how different algorithmic choices affect performance and where the performance benefits of fast connectivity implementations come from. Our paper provides the fastest GPU implementations of connectivity, which we obtain by combining many combinations of algorithmic choices that prior work did not explore. Building high-performance implementations of these optimizations and combining them is key to achieving high performance. We believe that our proposed techniques can be integrated into graph processing compilers and other frameworks to significantly improve performance.

The remainder of the paper is organized as follows. Section 2 discusses graph terminology and prior work. Section 3 describes the design choices in GConn, and their relation to prior work. We perform a comprehensive evaluation of our implementations in Section 4, and conclude in Section 5.

2 NOTATION, PRELIMINARIES, AND PRIOR WORK

Graph Notation and Formats. In this paper, we focus on undirected, unweighted graphs, which we denote as $G = (V, E)$. V and E are the set of vertices and edges, and $n = |V|$ and $m = |E|$ are the number of vertices and edges, respectively. We consider two graph formats, coordinate list (COO) and compressed sparse row (CSR), with 0-based index notation (i.e., each vertex is labeled with a unique identifier in the range $[0, \dots, n - 1]$). The COO format is represented using an array of edges, where each edge contains a source and a destination vertex. The CSR format contains two arrays, *Offsets* and *Edges*, of lengths $n + 1$ and m , respectively. The edges for vertex i are stored in $Offsets[i], \dots, Offsets[i + 1] - 1$, and $Offsets[n] = m$, and we assume they are in sorted order.

Graph Connectivity Problems. A *connected component (CC)* in G is a maximal set of vertices in which each pair of vertices in the component is connected by a path. A connected components algorithm computes a vertex label $labels(v)$ for each $v \in V$. Two vertices $u, v \in V$ are in the same component if and only if $labels(u) = labels(v)$. A *connectivity query* returns true if and only if two input vertices belong to the same component. A *spanning forest (SF)* maintains a connected tree for each connected component. A *breadth-first search (BFS)* takes a graph with a source vertex and returns an array A where $A(v)$ stores the shortest path between the source and v . If a vertex v is unreachable from the source vertex, $A(v)$ is set to infinity.

A *union-find* (disjoint set) data structure keeps track of a number of disjoint sets such that elements in each set have the same labels. Each set is represented by a tree and each element in the set has a parent pointer. The parent of a tree root points to itself. In this paper, we assume that elements either point to themselves or point to a parent with a smaller ID. The data structure provides three operations: MAKESET, UNION, and FIND [25]. MAKESET(u) generates a new tree that is composed of only one node u , which is a root. UNION(u, v) joins two trees, u and v , into a single tree. FIND(u) returns the label of the root of the tree containing u . FIND and UNION operations can perform path compression during execution to speed up subsequent operations. Union-find can be used for connectivity by creating a set for each vertex, and calling UNION on the endpoints of each edge. The labels of the vertices can be obtained in the end by running FIND on each vertex.

In this paper, we also consider connectivity algorithms that maintain trees, where we can move vertices between trees without completely joining the trees (as needed for merging sets in union-find). We refer to an algorithm as *root-based* if it only modifies the parent pointers of tree roots to point to vertices in other trees (vertices are still free to update their pointer to point to other vertices in the same tree). All of the union-find algorithms are root-based.

Compare-and-Swap. In this paper, we use the atomic *compare-and-swap* (CAS) primitive, which takes as input a memory location, an old value, and a new value. If the value stored in the memory location is equal to the old value, then the CAS atomically replaces the old value with the new value, and returns true. Otherwise, the CAS does not update the value, and returns false.

Prior work on GPUs. Soman et al. [82] provide GPU-CC, the first high-performance implementation of the Shiloach-Vishkin algorithm [77] on GPUs. Many libraries (e.g., Gunrock [89], IrGL [71], and Groute [13]) also adopt a variant of this approach. Note that GPU-CC uses the COO format so the performance does not suffer from poor load-balance on GPUs. GSWITCH [65] is a framework for graph processing which provides different combinations of optimization strategies for different graph algorithms, including connectivity. Hence, GSWITCH allows tens of different combinations of optimizations (e.g., different load-balancing strategies can be combined with other optimization strategies). The best set of optimizations is found using a machine learning approach. ECL-CC [49] uses a concurrent union-find algorithm for connectivity in the CSR format. Afforest [85] incorporates the k -out sampling strategy (described in Section 3.3) that significantly improves performance on many real-world graphs. Afforest uses the CSR format, and uses the load-balancing technique from [13].

3 GConn OVERVIEW

We first describe the overview of ConnectIt in Section 3.1. Then we provide an overview of the GConn framework in Section 3.2. In Sections 3.3 and 3.4, we provide an overview of specific algorithmic choices that we explore. Due to space constraints, we defer the complete description and pseudocode for some of these methods to the full version of our paper [45].

3.1 Overview of ConnectIt

ConnectIt is a framework for multicore connectivity algorithms that outperforms existing state-of-the-art multicore algorithms and generates various implementations for connectivity based on a two-phase execution model: a *sampling* phase and a *finish* phase. In the sampling phase, a subset of edges are inspected to partially form connected components. Next, the most frequently occurring label (L_{\max}) is identified. In the finish phase, only vertices whose label is not equal to L_{\max} need to process their outgoing edges. Vertices with label L_{\max} skip processing their edges, since any neighbor with label L_{\max} is already in the same component, and any neighbor with label other than L_{\max} will process an edge to this vertex. This two-phase execution can significantly reduce the number of edges processed. ConnectIt provides correctness proofs for the two-phase execution. GConn is an extension of ConnectIt that we developed for GPUs, which enables easy exploration of different algorithmic choices.

Algorithm 1 GConn for Static Connectivity

```

1: procedure STATICCONN( $G(V, E)$ ,  $sample$ ,  $finish$ ,  $compress$ )
2:    $labels \leftarrow \text{INITLABEL}(V)$ 
3:    $labels \leftarrow \text{SAMPLEPHASE}(G, labels, sample, finish, compress)$ 
4:    $L_{\max} \leftarrow \text{GETMOSTFREQUENTLABEL}(labels)$ 
5:    $labels \leftarrow \text{FINISHPHASE}(G, labels, finish, comp)$ 
6:    $labels \leftarrow \text{LABELFINALIZATION}(V, labels)$ 
7:   return  $labels$ 

```

Algorithm 2 GConn for Spanning Forest

```

1: procedure SPANNINGFOREST( $G(V, E)$ ,  $sample$ ,  $finish$ ,  $compress$ )
2:    $labels \leftarrow \text{INITLABEL}(V)$ 
3:    $edges \leftarrow \text{INITEDGE}(V)$ 
4:    $(labels, edges) \leftarrow \text{SAMPLEPHASE}(G, labels, sample, finish, compress)$ 
5:    $L_{\max} \leftarrow \text{GETMOSTFREQUENTLABEL}(labels)$ 
6:    $(labels, edges) \leftarrow \text{FINISHPHASE}(G, labels, finish, comp)$ 
7:   return  $edges$ 

```

3.2 GConn Framework

In this section, we give an overview of the GConn that we use to obtain our GPU implementations for static connectivity, spanning forest, and incremental connectivity.

Static Connectivity. Algorithm 1 presents the main steps in GConn. GConn takes in a graph in either CSR or COO format, as well as parameters for the sampling algorithm (*sample*), the finish algorithm (*finish*), and the compression algorithm (*compress*). The label of each vertex is initialized to its own ID on Line 2, and the sampling phase is performed on Line 3. On Line 4, the most frequently occurring label (L_{\max}) is identified from the result of the sampling phase. On Line 5, GConn performs the finish phase, processing edges of vertices with label not equal to L_{\max} . Finally, on Line 6, we finalize the labels of each vertex by assigning each vertex the label of the root of its tree. We use C++ templates and inlined functions to achieve high-performance implementations while keeping the GConn implementations high-level. Our implementations modularize the routines for the sampling algorithm, finish algorithm, compression algorithm, the load-balancing strategy, and graph format, making it easy to test different implementations and add new variants.

Static Spanning Forest. As shown in Algorithm 2, implementations for spanning forest are similar to those for connectivity; GConn supports different combinations of sampling and finish methods while generating correct spanning forest algorithms. In conjunction with the label initialization (Line 2), we also maintain the edges in the spanning forest using an auxiliary array of size n (Line 3). The key idea behind our implementations of spanning forest is to assign each edge of the discovered spanning forest to a unique vertex that is one of the endpoints of the edge. This special vertex is the root vertex in the union-find structure that updates its parent pointer to point to another vertex, so that it is no longer a root. Hence, the root-based algorithms for connectivity (to be described in Section 3.4) can be converted to compute spanning forests. The sampling phase simply creates a subset of the edges for the spanning forest (Line 4), while computing partially connected components and determining L_{\max} , as in static connectivity (Line 5). After that, using L_{\max} , the finish phase computes the rest of the edges for the spanning forest (Line 6). The finalization step is not required for spanning forest since the edges of the spanning forest have already been generated in previous steps, which eliminates the overhead for the post-processing. In our experiments, we found

that spanning forest algorithms are 6% faster on average than their static connectivity counterpart.

Incremental Connectivity. As many real-world graphs are being updated frequently, many connectivity algorithms have been proposed for dynamic graphs [1, 28, 30, 64, 76, 80]. Many of our algorithms are a natural fit for the *incremental* setting, where edges are inserted but not deleted. We designed GConn to support incremental connectivity (and spanning forest) algorithms that receive batches of operations consisting of edge insertions and connectivity queries that can be executed in parallel. We will describe how GConn supports incremental connectivity algorithms in Section 3.6.

3.3 Sampling Algorithms

As done in ConnectIt [29], we decompose connectivity algorithms into the sampling and finish phases. The sampling phase traverses a subset of edges in the graph to update the labels of vertices. The sampling phase can reduce the number of edges inspected in the finish phase, since in practice we expect that a large fraction of the vertices will already be settled in the L_{\max} component after applying the sampling phase. All connectivity algorithms in the literature today, except for Afforest [85], only support a finish phase. We implement sampling for graphs in CSR format due to the ease of skipping over all edges for particular vertices (i.e., the ones with label L_{\max} after sampling). Below we introduce the different sampling methods implemented in GConn. We discuss them in the context of connectivity, although they are used similarly in spanning forest. Pseudocode for these methods can be found in the full version of our paper [45].

k -out Sampling. Given a parameter k , *k -out sampling* computes connected components on a *sampled graph* constructed by uniformly sampling k edges out of each vertex [44]. Sutton et al. [85] use a type of k -out sampling strategy where the *first* k edges out of each vertex are used. The vertices obtain labels as a result of running a parallel connected components algorithm on the sampled graph. In practice, after applying k -out sampling, many of the vertices in the largest connected component will have the same label, since many real-world graphs have a single massive component that most vertices will be a part of in the sampled graph. In GConn, we implemented several variants of k -out sampling for different values of k , and found that taking the first 2 edges, as in [85], gave the best performance overall. Taking the first 2 edges per vertex enabled us to label most of the vertices in the largest component with the final label, and minimized the overall number of edge inspections during the sampling and finish phases. Moreover, since in many real-world graphs, the edges for a vertex are sorted and if all vertices choose their first two edges (neighbors), there are likely to be many shared neighbors, which improves locality and increases the size of the large component found. For running connectivity on the subgraph, we use different variants of union-find (discussed in Section 3.4).

Hook-Based Search (HB) Sampling. Inspired by ECL-CC [49], we designed the hook-based search (HB) sampling approach to potentially reduce the number of edge traversals compared to k -out sampling. This approach uses a union-find structure to maintain vertex labels. First, each vertex v inspects its smallest (i.e., first) neighbor w , and updates its label to $labels(v) = \min(labels(v), labels(w))$.

This step is efficient since there is no contention. Second, for all vertices that are still roots (i.e., $labels(v) = v$), we inspect their first N edges and apply the union operation to these edges. The goal of these steps is to minimize the total number of roots after sampling, since fewer roots mean that the graph is more connected. This approach can potentially require fewer edge traversals than k -out sampling for $k \geq 2$ if there are few roots after the first step.

Breadth-First Search (BFS) Sampling. In breadth-first search (BFS) sampling, we run a BFS from a chosen source vertex, which discovers the connected component of this source. If the graph has a massive connected component (containing a large fraction of the vertices), we have a high probability of finding the largest connected component. Connected component algorithms typically incur a large overhead in a concurrent setting, whereas BFS with idempotent operations can incur a smaller overhead [66]. Furthermore, the performance of BFS can be improved using direction-optimizing to reduce unnecessary edge traversals for many real-world graphs [9]. In our BFS sampling strategy, we applied a BFS from a source vertex, which we chose by sampling a subset of vertices and using the vertex with the largest degree from the sample. This idea was used by Slota et al. [81] to speed up label propagation.

3.4 Finish Algorithms

Here we introduce the finish algorithms in GConn. Again, we discuss them in the context of connectivity, but they are applied similarly in spanning forest. Our algorithms are min-based algorithms, where vertices maintain labels, propagate labels to other connected vertices, and keep the minimum label received. When the algorithms converge, vertices will have the same label if and only if they are in the same component. Pseudocode for our algorithms can be found in the full version of our paper [45].

Union-Find. Union-find algorithms are a special case of min-based algorithms that use a disjoint set data structure to maintain and propagate labels. GConn includes a broad set of concurrent union-find algorithms that are obtained by combining different union operations with different path compression strategies, all of which are root-based algorithms. In particular, it contains concurrent GPU implementations of union used in Rem's algorithm (Union-Rem-Lock [73]), randomized linking by index (Union-Async [52]), and randomized linking by rank (Union-JTB [53]). Note that Union-Async and Union-JTB are lock-free compare-and-swap (CAS) implementations, whereas Union-Rem-Lock is a lock-based implementation. Spin-locks are used in Union-Rem-Lock, which can significantly degrade parallelism on GPUs [32], so we also implemented a lock-free version using CAS (Union-Rem-CAS). We also implement two variations of Union-Async: Union-Early, which traverses the paths of the two inputs simultaneously and terminates once a common node is reached [52]; and Union-Hooks, which performs CAS operations on an auxiliary *hooks* array so that writes to the *labels* array are uncontended (on the other hand, Union-Async directly performs CAS operations directly on the *labels* array). All of our union operations link from larger to smaller ID to ensure that there are no cycles. FIND is implemented by traversing to the root of the tree of the input vertex.

In our implementations, path compression is done on-the-fly when calling either UNION or FIND, and GConn includes five options: no path compression, path-splitting, path-halving, full path compression, and path-splicing. We describe these operations in detail in the full version of our paper [45]. Each union operation can be combined with a subset of these path compression rules. ConnectIt[29] proves which of the combinations are valid, and GConn supports the valid combinations from [29].

All of the implementations are asynchronous, meaning that UNION and FIND calls can be executed concurrently without synchronization. Furthermore, all of the implementations, except for Union-Rem-Lock, are wait-free. As far as we know, none of the variants of union-find above have been implemented for GPUs in the literature. The only union-find variants that have been implemented on GPUs are ECL-CC [49] and Afforest [85]. ECL-CC [49] implements a union-find algorithm, which uses the Union-Async rule, but has a separate path compression step to fully compress the paths, which requires synchronization. Afforest [85] implements a variant of the classic Shiloach-Vishkin algorithm [77]. Their algorithm also has a separate compression step, and requires synchronization. Both ECL-CC and Afforest are root-based algorithms. We also implemented these algorithms using GConn.

Other Min-based Algorithms. Besides the union-find algorithms previously described, we also implement the following min-based algorithms in GConn: Shiloach-Vishkin (SV) algorithms [3], Liu-Tarjan (LT) algorithms [62], Stergiou’s algorithm [83], and the Label Propagation (LP) algorithm [78]. These other min-based algorithms generalize the union-find algorithms by allowing a vertex v in tree T to be moved to another tree T' , without requiring that all vertices in T be moved to T' . The algorithms still link from larger to smaller ID to prevent cycles.

The SV algorithm is a classical parallel connectivity algorithm, and many variants of it have been proposed in the literature, e.g., [3, 5, 10, 13, 24, 36, 65, 71, 82, 89, 92]. Besides Afforest, all other GPU implementations [13, 65, 71, 82, 89] are based on a GPU implementation by Soman et al. [82]. Soman et al.’s algorithm alternates between hooking from smaller to larger ID and from larger to smaller ID (the idea was originally proposed by Greiner [36]), but does not guarantee that only roots are hooked. We also implemented a root-based variant of SV, where on every round each vertex uses an atomic minimum operation to update their neighbors’ labels, followed by full path compression via pointer jumping.

The LT algorithms are generated by combining several simple rules about how to update the *labels* array by using edges to transfer connectivity information. The updates are done using atomic min operations. In their paper [62], only five algorithms are considered, but ConnectIt supports 16 variants by considering more combinations of rules (all are min-based, and 6 variants are root-based as well). Stergiou’s algorithm [83] is very similar to one of the LT variants, but it maintains two *labels* arrays, one for the previous iteration and one for the current iteration.

Many implementations and frameworks for connectivity adopt the LP algorithm, including [69, 78, 81]. In each round of the LP algorithm, the labels corresponding to the endpoints of each edge are compared, and if they are different, the larger label updates

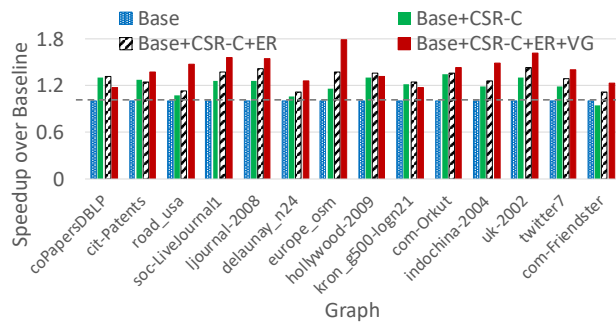


Figure 2: The speedup of adding each optimization technique against Baseline for Union-Async with path compression and k -out sampling, which is the fastest connectivity variant on average. The performance of each graph is normalized to Baseline.

itself to be equal to the smaller label. The LP algorithm terminates when there is a round in which no labels are updated.

3.5 Iterative GPU Optimizations

This section describes three key GPU optimizations—*CSR coalescing* (CSR-C), *edge reorganization* (ER), and *vertex gathering* (VG)—that we apply to our implementations in GConn. For practical purposes, we only consider the optimization strategies that improve the performance of the fastest GConn implementations. Since we found Afforest [85] to be the fastest existing implementation in most cases, we identify the performance bottlenecks in Afforest, and show how to iteratively apply our optimizations to achieve the performance of our fastest implementations. We first substituted their union-find implementation with Union-Async, which we found to be faster, and use this as the baseline. Figure 2 shows how performance improves over the baseline (*Base*) with each additional optimization applied.

CSR Coalescing (CSR-C). We identified that the sampling phase requires a significant amount of time in Afforest (35.15–98.63%, with a median of 88.80%), as shown in our experimental evaluation in Section 4.2. During k -out sampling with $k = 2$, Afforest has two sub-phases, the first which traverses the first edge out of every vertex, and the second which traverses the second edge out of every vertex. This strategy traverses the scattered entries in the CSR array twice, which wastes memory bandwidth. Since the first two edges of a vertex are contiguous in the memory layout, the two edges of a vertex can be processed simultaneously by two adjacent threads, which makes memory accesses for the CSR array coalesced and halves the data volume needed to access the CSR array. We observe that applying this optimization on top of the baseline (*Base+CSR-C* in Figure 2) improves performance by an average of 20.68%.

Edge Reorganization (ER). During k -out sampling, we found that CAS operations make the warp-efficiency on the GPU very low due to only a few threads in a warp being active. For these algorithms, if two edges having the same endpoint try to update a memory location at the same time, then the updates can become serialized. This is often the case in CSR format since edges incident to a single vertex are processed in the same warp. To alleviate this issue, we could have each thread read the first two edges of a

vertex and process them sequentially; however, this would cause threads to have scattered accesses to the CSR edge array, which decreases memory efficiency. The edge reorganization optimization groups threads into pairs, where each pair reads the two edges for one vertex together, and then reads the two edges for another vertex together. Each time, the pair of threads reads contiguous locations in memory. Then, using warp shuffling [18, 84] the four edges can be reorganized so that the edges from the same vertex are contiguous in memory, enabling each thread to process the two edges of a vertex serially to avoid CAS operations. *Base+CSR-C+ER* in Figure 2 shows the performance improvement after applying ER on top of CSR-C. We achieve an average performance improvement of 28.79% over the baseline.

Vertex Gathering (VG). The previous optimizations improve the performance of the sampling phase. However, we found that the performance of the finish phase can also be improved. In the kernel for the finish phase in Afforest, each thread accesses entries in the *labels* array in a cyclic fashion, and if the entry of the i 'th label is not equal to L_{\max} , then the edges of vertex i are processed in a load-balanced fashion by distributing the edges across threads. However, as we show in Section 4.2, only a small subset of vertices have labels not equal to L_{\max} . Therefore, many threads will be idle during the finish phase, which degrades performance. To improve performance in the finish phase, we first aggregate the vertices whose labels are not equal to L_{\max} into an *active vertex set*, and in the finish phase, each thread reads a vertex in the active vertex set, and distributes the edges for load-balancing. We show the performance improvement of adding the vertex gathering optimization in *Base+CSR-C+ER+VG* of Figure 2. With all three optimizations, the overall performance is 41.55% faster than the baseline on average.

3.6 Incremental Connectivity Support

Algorithm 3 GConn for Incremental Connectivity

```

1: procedure INCREMENTAL( $G(V, E)$ , sample, finish, compress)
2:   labels  $\leftarrow$  STATICCONN( $G(V, E)$ , sample, finish, compress)
3:   for  $B \in$  Batches do
4:     (labels, query_results)  $\leftarrow$ 
       FINISHPHASEBATCH( $B$ , labels, finish, comp)

```

This section describes how GConn supports incremental connectivity given batches of updates. We support incremental connectivity for the root-based algorithms, and the pseudocode is shown in Algorithm 3. We first generate the initial graph (which can be empty) by initializing the *labels* array for the vertices in the graph (Line 2) using the STATICCONN procedure from Algorithm 1. We assume that the *labels* array is large enough to hold all of the vertices that will be encountered, but we only initialize the entries for vertices present in the initial graph. Batches of insertions and connectivity queries arrive in an online fashion, and are processed using one of the finish algorithms on just the batch (Lines 3–4). The batches are given in COO format, which is a natural input for streaming algorithms. However, sampling is inefficient for graphs in COO format, and hence we do not use sampling for incremental algorithms.

Each batch B is composed of a mix of inserts and queries. For an update, FINISHPHASEBATCH updates the *labels* array as FINISHPHASE from Algorithm 1 does. Furthermore, a thread that inserts a new vertex will initialize its entry in the *labels* array by acquiring

a spin lock. For a query, FINISHPHASEBATCH calls the FIND function (one of the compression algorithms GConn provides) for both query endpoints to check whether they are in the same component. The query results (*query_results* in Line 4) are stored in a bitvector: the i 'th entry in the bitvector is true if the i 'th edge of B is a query, and the two endpoints of it are in the same component.

As shown in Algorithm 3, the previous batches are never inspected because the root-based algorithms guarantee correctness *without* requiring inspecting edges in a previous batch [29]. The root-based algorithms incorporated into GConn are all union-find algorithms used for static connectivity, Shiloach-Vishkin, and the root-based Liu-Tarjan algorithms. However, when Union-Rem-CAS and Union-Rem-Lock with SpliceAtomic is used, the updates and queries need to be processed separately to guarantee correctness [29].

4 EVALUATION

In this section, we provide an experimental evaluation and analysis of GConn. All numbers reported in this section are the median of five runs on the Volta machine unless noted otherwise. We found that the trends for spanning forest are similar to the trends for connectivity, with our spanning forest implementations obtaining an average speedup of 6% over the connectivity implementations due to not requiring the label finalization step.

Overview of Results. The results of this section can be summarized as follows:

- We provide an experimental evaluation of GConn connectivity implementations in the no sampling setting (Section 4.1). Without sampling, Union-Async and Union-Rem-CAS are the fastest implementations.
- In the sampling setting, we provide a detailed analysis of different sampling procedures and find that k -out sampling or HB sampling can significantly improve the performance unless the average degree of vertices is low (Section 4.2).
- The fastest GConn algorithms consistently and significantly outperform state-of-the-art GPU connectivity implementations (Section 4.3).
- GConn incremental connectivity algorithms can achieve a throughput of tens of billions of edges per second. We also evaluate the throughput for different batch sizes, and ratios of insertions to queries (Section 4.4). In the incremental setting, Union-Async is usually the fastest implementation.
- Compared to ConnectIt, a framework for CPU connectivity algorithms, GConn achieves 8.26–14.51x speedup for static connectivity and 1.85–13.36x speedup for incremental connectivity. Our analyses also show GPUs are an attractive platform for connectivity algorithms in terms of both speed and cost (Section 4.6).

Experimental Setup. Our GPU evaluation is performed on two machines. The first is an NVIDIA Tesla V100, which is a Volta generation GPU with 32GB that offers a 900 GB/sec memory bandwidth, 6MB of L2 cache, and 128KB of L1 cache per Streaming Multiprocessor (SM) with a total of 80 SMs. The second is an NVIDIA TITAN Xp, which is a Pascal generation GPU with 12GB that offers a 547.6 GB/sec memory bandwidth, 3MB of L2 cache, and 48KB of L1 cache per SM with a total of 30 SMs. All implementations are compiled with NVCC v10.0 using the `-O3` and `--use_fast_math` flags.

Dataset	n	m	Diam.	Num. Comps.	Largest Comp.
coPapersDBLP	540.49K	30.49M	15*	1	540.49K
cit-Patents	3.77M	33.04M	20*	3,627	3.76M
road_usa	23.95M	57.71M	6,809	1	23.95M
soc-LiveJournal1	4.85M	85.70M	16	1,876	4.84M
ljournal-2008	5.36M	99.03M	31*	75	5.36M
delaunay_n24	16.78M	100.66M	1,720*	1	16.78M
europa_osm	50.91M	108.11M	19,314*	1	50.91M
hollywood-2009	1.14M	112.75M	11	44,508	1.07M
kron_g500-logn21	2.10M	182.08M	6	553,159	1.54M
com-Orkut	3.07M	234.37M	9	1	3.07M
indochina-2004	7.41M	301.97M	26	295	7.32M
uk-2002	18.52M	523.57M	29*	38,359	18.46M
twitter7	41.65M	2.41B	23*	1	41.65M
com-Friendster	65.61M	3.61B	32	1	65.61M

Table 1: Graph inputs, including number of vertices (n), edges (m), diameter, number of connected components, and the largest connected component. The graphs are symmetric, and edges are counted once in each direction. For graphs on which we were unable to compute the exact diameter, we compute the effective diameter (marked with *), which is a lower bound on the actual diameter.

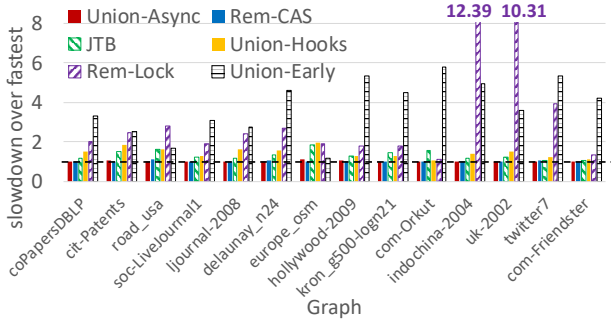


Figure 3: The slowdown over the fastest union-find variants for each graph in the no-sampling setting.

Graph Data. To show how GConn performs on various graphs of different scales, we selected all publicly-available graphs used in ECL-CC [49], EvoGraph [76], and ConnectIt [29] that have more than 30M edges and fit in the GPU memory. Table 1 shows the details of our graph inputs, including the number of vertices and edges, the graph diameter, the number of connected components, and the size of the largest component. Our inputs include many Web and social network graphs that have low diameters, as well as road networks that have high diameters. All graphs that we use were obtained from SuiteSparse [26] or SNAP [60]. We symmetrized all of the graphs.

4.1 Static Parallel Connectivity without Sampling

In this section, we evaluate our GConn implementations for static connectivity algorithms in the *No Sampling* setting.

Evaluation of Union-Find Variants. The first group of rows in Table 2 shows the results of the fastest implementations of each algorithm in the no sampling setting.

Figure 3 shows the slowdown of the fastest of each of the union-find variants over the fastest overall variant for each graph. For each graph, six bars are listed in order of average performance.

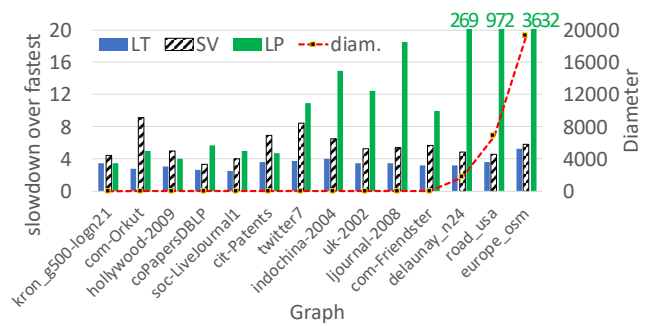


Figure 4: Slowdowns for other min-based algorithms compared to the fastest union-find variants for each graph in the no-sampling setting. The graphs are sorted in ascending order of diameter.

From Figure 3, we observe that the fastest implementation is either Union-Async or Union-Rem-CAS in the *No Sampling* setting. Union-Rem-CAS is 1.02x slower on average than Union-Async across all graphs, and Union-JTB is 1.26x slower on average than Union-Async due to the usage of 64-bit CAS operations. Union-Hooks is designed to reduce the overhead for atomic operations, but it is 1.44x slower on average than Union-Async due to a costly memory barrier needed to avoid race conditions. Union-Rem-Lock is 3.81x slower on average than Union-Rem-CAS due to the poor performance of spin locks on GPUs.

Evaluation of Other Min-based Algorithms. Figure 4 shows the slowdown of the other min-based algorithms compared to the fastest variant across all algorithms in the no-sampling setting. The graphs that are sorted in order of increasing diameter (presented with the red curve).

As shown in Figure 4, the other min-based algorithms are much slower than the union-find algorithms because union-find algorithms only inspect each edge at once, whereas the other algorithms typically traverse edges multiple times, which leads to redundant computation. The fastest Liu-Tarjan and Shiloach-Visshin variants are 3.72x and 5.19x slower on average than Union-Async in the no-sampling setting, respectively. We have included the Stergiou algorithm in the category of Liu-Tarjan algorithms as this algorithm is similar to Liu-Tarjan algorithms; however, it is always much slower than the fastest variant of Liu-Tarjan algorithms (up to 66x slower).

As shown for the three graphs located on the far right in Figure 4, the performance of LabelPropagation degrades significantly as the diameter increases. This is because LabelPropagation needs a large number of rounds to propagate the minimum label to all vertices, and during that time most vertices are active. Even on low-diameter graphs (diameter at most 32 in our experiments), LabelPropagation is still 7.76x slower on average than the union-find algorithms since LabelPropagation still requires multiple traversals per edge, whereas the union-find algorithms do not.

As shown in Table 2, we also incorporate the finish algorithms used in two fastest state-of-the-art works, ECL-CC [49] (denoted as G_ECL-CC) and Afforest [85] (denoted as G_Afforest), into GConn, and evaluate them in Section 4.3.

Group	Algorithm	coPapers DBLP	cit- Patents	road_usa	soc-Live Journal1	ljournal -2008	delaunay _n24	europa _osm	hollywood -2009	kron_g500 -logn21	com- Orkut	indochina -2004	uk-2002	twitter7	com- Friendster
No sampling	Union-Early	1.49	4.79	5.68	8.18	8.83	17.65	6.87	8.14	16.23	14.11	26.39	32.89	375.50	853.87
	Union-Hooks	0.68	3.44	5.56	3.94	5.26	5.98	11.67	1.95	4.65	4.02	7.36	13.97	155.98	410.61
	Union-Async	0.46	2.02	3.39	3.07	3.17	3.84	6.76	1.62	3.64	3.85	5.32	9.09	123.78	365.98
	Union-Rem-CAS	0.45	1.88	3.83	2.98	3.20	4.09	5.90	1.53	3.60	3.70	5.39	9.26	133.46	369.59
	Union-Rem-Lock	0.90	4.66	9.57	5.73	7.59	10.31	11.45	2.73	6.52	4.19	65.96	93.69	484.23	487.72
	Union-JTB	0.53	2.84	5.53	3.70	3.68	5.29	10.85	1.96	5.23	5.78	6.40	11.07	130.01	400.17
	Liu-Tarjan	1.20	6.73	12.13	7.72	11.27	12.60	31.23	4.69	12.39	10.16	21.53	31.44	464.76	998.77
	Shiloach-Vishkin	1.47	13.05	15.65	12.23	17.17	18.63	34.49	7.57	15.93	34.09	35.01	47.73	1056.17	2066.00
	LabelPropagation	2.57	8.98	3289.59	15.01	58.89	1032.09	2.14e4	6.18	12.50	18.58	79.25	112.74	1361.37	3646.96
	G_ECL-CC	0.51	2.26	3.76	3.18	3.80	4.19	10.09	1.92	3.84	5.01	6.75	11.58	161.13	385.48
G_Afforest	5.96	23.07	4.01	25.46	42.12	6.70	19.07	85.02	91.66	97.24	43.12	73.80	143.75	1661.75	
k-out sampling	Union-Early	0.26	3.30	8.93	1.28	1.85	6.34	9.50	0.74	0.85	1.00	7.33	7.83	22.59	49.34
	Union-Hooks	0.20	2.52	8.13	1.05	1.63	4.87	10.57	0.39	0.61	0.74	2.42	4.39	14.01	31.09
	Union-Async	0.18	1.60	5.27	0.85	1.19	3.60	6.95	0.35	0.51	0.63	2.07	3.46	10.91	27.79
	Union-Rem-CAS	0.16	1.98	5.98	0.86	1.38	3.44	7.78	0.33	0.53	0.62	1.86	3.52	12.81	44.87
	Union-Rem-Lock	0.71	3.79	13.28	1.80	3.00	8.46	13.57	0.98	0.99	0.74	35.74	41.10	26.83	53.61
	Union-JTB	0.24	2.31	6.97	1.27	1.75	4.66	13.38	0.47	0.65	0.86	2.95	5.04	13.80	32.20
	Liu-Tarjan	1.29	11.56	20.71	3.07	13.96	15.19	40.17	5.20	13.05	0.65	20.20	26.76	41.69	522.27
	Shiloach-Vishkin	1.55	18.88	21.35	12.68	17.37	17.42	47.61	8.46	15.99	33.79	32.47	67.94	1150.68	2059.18
	LabelPropagation	2.28	16.44	8228.75	2.67	72.57	895.65	65136.83	8.32	18.65	0.60	103.27	134.05	27.01	163.80
	G_ECL-CC	0.19	1.67	4.88	0.88	1.28	3.19	7.28	0.38	0.56	0.66	2.11	3.80	11.14	27.29
G_Afforest	0.24	4.72	5.63	1.25	4.40	3.81	9.58	0.84	2.41	0.80	2.79	5.60	13.46	31.84	
HB sampling	Union-Early	0.41	2.59	17.79	1.03	1.86	48.33	25.42	1.48	0.76	0.49	12.95	17.26	12.62	19.26
	Union-Hooks	0.24	1.75	12.45	0.70	1.41	7.37	16.78	0.43	0.64	0.50	3.00	5.99	9.90	17.97
	Union-Async	0.24	1.47	8.60	0.67	1.22	5.95	14.99	0.41	0.57	0.49	2.60	4.97	9.12	17.75
	Union-Rem-CAS	0.24	1.56	8.62	0.67	1.28	6.37	13.66	0.41	0.57	0.50	2.43	4.83	9.60	18.30
	Union-Rem-Lock	1.51	2.39	14.37	8.16	1.90	9.63	17.11	0.60	1.59	0.49	10.60	20.95	11.78	19.09
	Union-JTB	0.70	5.68	8.54	6.09	5.37	7.09	18.86	2.56	11.11	15.28	9.78	12.13	339.15	738.97
	Liu-Tarjan	1.36	11.68	21.85	1.91	10.53	24.67	40.01	3.70	9.06	0.61	15.80	27.15	14.33	353.80
	Shiloach-Vishkin	2.94	18.55	21.38	6.33	14.49	24.37	54.87	9.20	27.01	0.64	25.44	67.48	46.30	829.49
	LabelPropagation	6.58	16.62	7943.94	2.22	73.01	982.00	6.65e04	8.24	19.12	0.57	97.24	125.14	24.71	123.77
	G_ECL-CC	0.26	1.39	8.44	0.71	1.23	6.32	14.21	0.47	0.61	0.50	2.71	5.28	8.78	17.57
G_Afforest	0.26	2.85	8.04	0.73	2.03	9.12	13.80	0.55	1.14	0.50	3.56	6.22	10.83	17.75	
BFS sampling	Union-Early	1.34	2.39	409.15	2.10	2.67	90.27	1778.75	1.25	0.81	1.06	80.15	35.55	22.20	20.58
	Union-Hooks	1.35	2.38	391.73	1.90	2.68	92.41	1652.54	1.22	0.80	1.03	79.23	32.24	22.19	20.56
	Union-Async	1.32	2.40	390.92	1.93	2.68	92.91	1578.57	1.23	0.83	1.05	80.58	32.54	22.22	20.54
	Union-Rem-CAS	1.34	2.38	397.09	2.07	2.78	90.25	1598.75	1.26	0.82	1.03	79.64	32.32	22.16	20.52
	Union-Rem-Lock	1.32	2.38	399.62	2.06	2.74	92.41	1584.45	1.26	0.80	1.02	83.84	33.09	22.19	20.52
	Union-JTB	1.36	2.53	406.20	2.21	3.06	104.46	1627.93	1.29	0.89	1.18	80.51	32.89	23.98	23.72
	Liu-Tarjan	1.38	3.14	395.90	2.55	2.92	91.79	1659.59	1.44	1.04	1.10	81.54	35.14	22.57	21.19
	Shiloach-Vishkin	1.41	3.07	401.61	2.50	2.90	97.07	1616.75	1.53	1.05	1.15	82.49	34.78	23.29	22.19
	LabelPropagation	1.39	3.23	403.12	2.49	2.77	93.34	1626.54	1.51	0.95	1.07	89.03	40.99	22.35	20.86
	G_ECL-CC	1.33	2.41	397.33	2.06	2.82	94.21	1631.24	1.24	0.83	1.06	81.36	32.57	22.20	20.51
G_Afforest	1.30	2.39	393.34	2.06	2.80	91.45	1625.83	1.30	0.83	1.06	83.19	32.84	22.18	20.48	
Existing algorithms	GPU-CC [82]	2.77	9.74	22.85	10.18	31.5	22.83	46.38	21.38	34.04	21.25	44.96	49.01	417.25	x
	GSWITCH [65]	1.07	4.5	21.03	7.06	9.24	8.22	38.39	6.82	7.45	7.54	13.46	26.1	x	x
	ECL-CC [49]	0.78	5.89	22.55	7.14	7.71	18.94	25.97	2.1	6.02	8.58	9.06	26.31	258.56	415.44
	Afforest [85]	0.35	6.13	5.54	1.27	5.07	3.69	9.13	1.06	2.97	0.82	4.55	7.67	13.5	32.4
	BFS-CC	1.33	721.59	577.38	277.35	8.51	89.33	1749.63	4267.22	5.07e04	0.96	133.77	3698.56	21.07	19.22

Table 2: Running times of implementations in GConn and state-of-the-art static connectivity algorithms in milliseconds on a V100 GPU. We report running times for five groups: implementations with *No sampling*, *k-out sampling*, *HB sampling*, *BFS sampling*, and *existing algorithms*. Within each of the first four group, we display the fastest variant for each graph in green. For each graph, we also display the fastest variant across all groups in bold font. x means that we were unable to obtain results due to the graph not fitting in the GPU memory for the given implementation.

4.2 Static Parallel Connectivity with Sampling

This section studies how our three sampling strategies affect performance, in terms of their execution time and the quality of the resulting sub-problem that they generate for the finish step in GConn. We start by studying how *k-out sampling* performs.

Evaluation of *k-out sampling*. The second group of rows in Table 2 presents the results of the fastest implementations of each algorithm using *k-out sampling*. The fastest implementation is Union-Async, Union-Rem-CAS, or G_ECL-CC; Union-Rem-CAS

is 1.03x slower on average than Union-Async, and we defer the analysis for G_ECL-CC to Section 4.3.

Figure 5 shows how each sampling algorithm improves performance. To see the performance improvement with respect to the average degree of the vertices (i.e., m/n), we sorted the graphs in ascending order of the average degree.

For the union-find variants, except on the two road-network graphs (*road_usa* and *europa_osm*), *k-out sampling* improves performance over the unsampled versions by 6.16x on average due to the significant reduction of edges that need to be inspected, as we will show shortly. For the other min-based algorithms, other

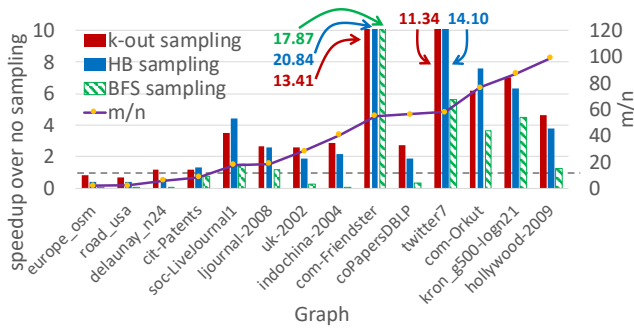


Figure 5: The speedup of the best variant of each sampling algorithm over the best variant in the no-sampling setting. The graphs are sorted in ascending order of m/n (i.e., the average degree).

than on the two road-network graphs, k -out sampling improves performance by 4.61x on average over the unsampled versions.

As shown in Figure 5, when the average degree is small (e.g., *road_usa* and *europa_osm*), k -out sampling as well as other sampling algorithms degrades performance. This is because most of the edges in the graph get inspected in the sampling phase, and there is additional overhead to split the computation across two phases. In general, k -out sampling decreases the number of edge inspections by up to $m - kn$, and so as the average degree increases, we can expect a higher potential performance improvement. As shown in Figure 5, k -out sampling usually shows better performance when the average degree is large.

Evaluation of HB sampling. The third group of rows in Table 2 presents the fastest variants of each algorithm using HB sampling with a default value of $N = 4$, which we found to work the best on average across our input graphs.¹ Unlike k -out sampling, HB sampling does not always improve performance; in many cases, the performance is degraded. The overall trend of HB sampling is similar to that of k -out sampling. As shown in Figure 5, when the average degree of vertices is small, HB sampling significantly degrades performance, and otherwise HB sampling can greatly improve performance. We will discuss how HB sampling compares with our other sampling schemes shortly.

Evaluation of BFS sampling. The fastest variants using BFS sampling are presented in the fourth group in Table 2. The effectiveness of BFS sampling is dependent on the diameter of an input graph and the parameters for the direction-optimizing BFS. As shown in Figure 5, for the high-diameter graphs (the three graphs on the far left), BFS sampling degrades performance by 24.59x on average over the unsampled versions due to the very small active vertex set for each BFS iteration and since a GPU kernel must be launched for each iteration. For the other graphs, BFS sampling achieves a 0.07–174.87x speedup over the unsampled versions. Note that as shown in the last row of Table 2, we also implemented the BFS-CC algorithm introduced in Ligra [78], in which a BFS is repeated on each new component until all components are found. BFS-CC performs

¹Although the optimal value of N can vary based on the input graph, determining the optimal value on a per-graph basis would incur significant overhead, which would outweigh the benefits of using the optimal parameter as opposed to the default parameter.

Graph	BFS Time	BFS Cov	BFS IC	KOut Time	KOut Cov	KOut IC	HB Time	HB Cov	HB IC
coPapersDBLP	60.3%	100.0%	0.0%	46.7%	98.9%	0.2%	97.2%	89.1%	5.4%
cit-Patents	83.9%	99.7%	0.0%	90.1%	98.2%	0.6%	99.5%	96.0%	2.3%
road_usa	86.8%	100.0%	0.0%	85.4%	95.9%	3.6%	83.1%	0.0%	100.0%
soc-LiveJournal1	86.9%	99.9%	0.0%	91.0%	99.9%	0.0%	92.1%	99.5%	1.1%
ljournal-2008	92.1%	100.0%	0.0%	83.7%	99.3%	0.2%	92.0%	94.9%	2.5%
delaunay_n24	92.3%	100.0%	0.0%	76.9%	100.0%	0.0%	89.3%	0.0%	100.0%
europa_osm	92.4%	100.0%	0.0%	79.2%	100.0%	0.0%	92.4%	0.0%	100.0%
hollywood-2009	55.1%	93.8%	0.1%	42.0%	91.0%	0.5%	99.8%	87.4%	2.0%
kron_g500-logn21	93.5%	73.6%	0.0%	41.2%	73.6%	0.0%	99.7%	73.5%	0.0%
com-Orkut	74.4%	100.0%	0.0%	54.4%	100.0%	0.0%	97.8%	100.0%	0.0%
indochina-2004	90.3%	98.7%	1.3%	42.5%	86.8%	7.4%	99.5%	63.2%	24.4%
uk-2002	95.9%	99.7%	0.1%	90.2%	92.0%	4.9%	88.2%	72.4%	20.8%
twitter7	93.4%	100.0%	0.0%	39.4%	100.0%	0.0%	99.7%	99.5%	0.0%
com-Friendster	98.0%	100.0%	0.0%	95.2%	100.0%	0.0%	90.3%	99.9%	0.0%

Table 3: This table presents how effective the sampling strategy is for each of our graph inputs. The Time columns show the percentage of sampling time to the total execution time. The Cov columns show the percentage of vertices that are in the largest connected component after the sampling phase. Hence, the edges incident to these vertices are not inspected in the finish phase. The IC columns show the percentage of inter-component edges that will be processed in the finish phase.

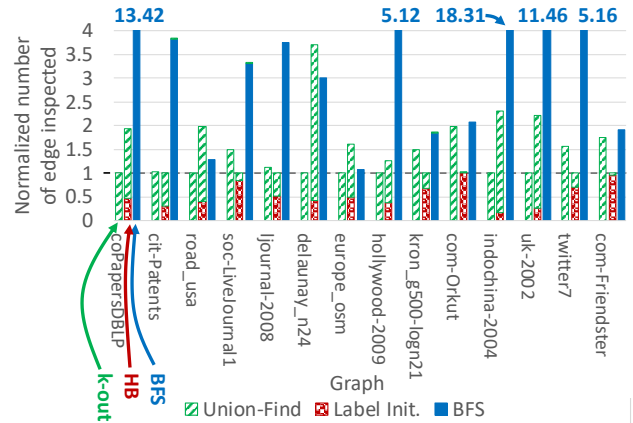


Figure 6: The normalized number of edges inspected for each sampling strategy. The first bar for each graph (green only) corresponds to the number of edge inspections for k -out using union-find. The second bar for each graph (red and green) shows the number of edge inspections in the label initialization step, and the number of edges inspected by the union-find step of HB sampling respectively. Finally, the blue bar shows the number of edges inspected in BFS sampling.

poorly due to the GPU kernel launch overhead for high-diameter graphs and for graphs with many connected components.

Comparing Different Sampling Strategies. Table 3 shows how many vertices and edges are covered by each sampling algorithm. As shown in Table 3, since BFS traverses all vertices connected to a source vertex likely to be in the largest connected component, the fraction of vertices covered (**Cov**) and the fraction of remaining

inter-component edges (**IC**) are maximized and minimized, respectively. Unfortunately, as shown in Table 2, BFS sampling takes significantly longer than other sampling algorithms, especially for the high-diameter graphs (*delaney_n24*, *road_usa*, and *europa_osm*) due to the kernel launch overhead and low parallelism on these graphs. It also inspects significantly more edges than the other two schemes, as we will show shortly. Although BFS sampling usually takes longer than k -out sampling, we observe that **CoV** and **IC** for BFS sampling and k -out sampling are very similar on our inputs. The sampling time for HB sampling is much lower than for the others due to the reduced number of edge inspections, but for some graphs, **CoV** and **IC** are significantly lower than for the other two schemes (especially for high-diameter graphs). The **Time** columns in Table 3 show the ratio of the sampling time to the total execution time. We see that the sampling phase takes most of the time, and hence, the design of an efficient, high-quality sampling algorithm is of paramount importance.

Figure 6 shows the normalized total number of edges inspected during the execution of each sampling algorithm. For each graph, Figure 6 shows three bars. The first bar (green only) corresponds to the number of edges inspected by k -out sampling using union-find. The second bar (red and green) shows the number of edges inspected during the label initialization step, and the union-find step of HB sampling, respectively. Finally, the third bar (blue) shows the number of edges inspected by BFS sampling. From Table 2 and Figure 6, we observe that BFS sampling is never the fastest, and HB sampling is the fastest when the number of edge inspections for union-find is small during the sampling phase. In this case, the total number of edge inspections is also minimized. For *com-Friendster*, as shown in Table 2, BFS sampling is faster than k -out sampling; the number of edge inspections with BFS sampling is usually the highest, but for *com-Friendster*, the total number of edge inspections for both BFS sampling and k -out sampling is similar, and thus BFS sampling is faster because the edge inspection step performed by the BFS (a single CAS on one of the endpoints) is much faster than the edge inspection by a union-find algorithm (a loop that must potentially run multiple times due to contention). As seen in Figures 5 and 6, for some graphs (e.g., *indochina-2004*, which has a speedup of 0.07x), the number of edge inspections is much higher for BFS, leading to significant performance degradation.

4.3 Comparison with State-of-the-art

This section compares our implementations with current state-of-the-art connectivity implementations on GPUs, which are shown in the last group of rows in Table 2. Figure 7 presents the normalized execution time over the fastest current state-of-the-art implementation for each graph with our fastest variant without and with sampling. Note that we did not report results for Groute [13], Gunrock [88], and IrGL [71] as they are outperformed by ECL-CC [49], which we compare against. We also tried the strategy by Cong and Muzio [24], but it never gave the best performance.

GPU-CC [82] and GSWITCH [65]. GPU-CC adopts a classic SV algorithm, which turns out to be slow as shown in Figure 7. Our fastest implementations without sampling and with sampling are 7.06x and 26.68x faster on average than GPU-CC, respectively. GSWITCH also applies a classic SV algorithm, but outperforms GPU-CC due

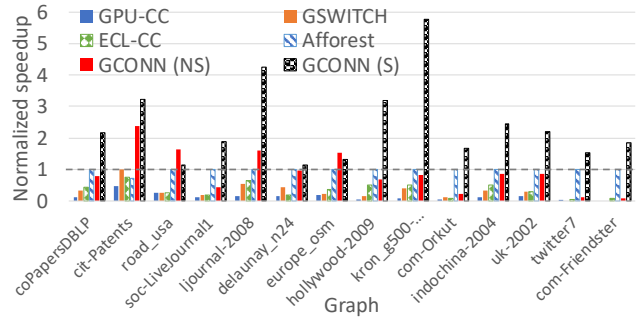


Figure 7: The normalized speedup over GPU-CC of current state-of-the-art implementations/libraries and GConn (with and without sampling). GCONN(NS) and GCONN(S) show the normalized speedup without and with sampling, respectively.

to several additional optimizations. Our fastest implementations without sampling and with sampling are 3.24x and 8.88x faster on average than GSWITCH.

ECL-CC [49]. ECL-CC is faster than both GPU-CC and GSWITCH due to using a more efficient algorithm based on union-find. As shown in Figure 7, our fastest implementations without sampling and with sampling are 2.78x and 10.15x faster on average than ECL-CC, respectively, mainly because we use a more efficient load-balancing strategy, and more efficient find and compress rules.

We also implemented the finish algorithm G_ECL-CC that is used in ECL-CC. As shown in Table 2, G_ECL-CC without sampling is 2.30x faster on average than ECL-CC. The speedup comes from using a different load-balancing strategy; in ECL-CC, one thread handles all edges of a vertex with degree at most 16, which can lead to significant load-imbalance. In contrast, GConn enables those edges to be processed in a load-balanced fashion using a variation of a strategy by Merrill et al. [66] that we designed. Without sampling and with sampling, G_ECL-CC is 1.19x and 1.04x slower on average than our fastest variants, respectively.

Afforest [85]. Afforest also implements a root-based algorithm with k -out sampling. As shown in Figure 7, our fastest implementations without sampling are 1% slower on average than Afforest with sampling due to the fact that many edges are not inspected in Afforest. Our fastest implementations with sampling are 2.51x faster on average because variants using HB sampling sometimes outperform those using k -out sampling, and the finish algorithm used in Afforest is slower than those used in our fastest variants. We also implemented $G_Afforest$, the finish algorithm used in Afforest, in GConn. We found that compared to other finish algorithms in the no sampling setting, $G_Afforest$ is 12.6x slower on average than our fastest finish algorithm. The main reason seems to be due to some union-find algorithms, such as Union-Async and Union-Rem-CAS, handling path compression more efficiently than the method used in $G_Afforest$.

4.4 Incremental Parallel Graph Connectivity

In this section, we evaluate incremental connectivity algorithms on GPUs. We achieve a raw speedup of 2,482x over EvoGraph [76] which is the fastest current state-of-the-art streaming connectivity implementation. Furthermore, the memory bandwidth-normalized

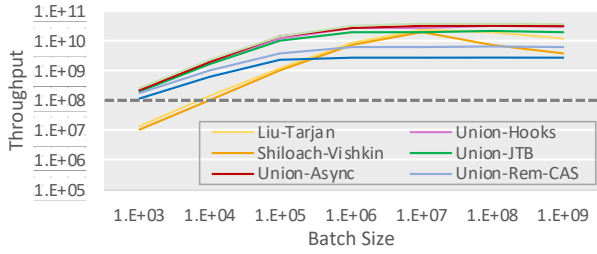


Figure 8: Throughput vs. batch size for *com-Orkut*.

speedup is 794x over EvoGraph. Unfortunately, we are not able to directly compare our streaming connectivity implementations with existing implementations on the same machine, although we provide indirect comparisons based on the numbers provided by the authors in their paper.² *cuSTINGER* [35] and *Hornet* [16] are GPU frameworks for streaming graphs, but their code does not contain streaming connectivity algorithms.

We conduct two types of streaming experiments. In the first type of experiment, we generate a stream of edge updates from the input graphs in Table 1. The second type of experiment models real-world graph streams using synthetic graph generators. Specifically, we use the RMAT generator [19] with parameters $(a, b, c) = (0.5, 0.1, 0.1)$ and the Barabasi-Albert (BA) generator [8]. For both generators, we use $n = 2^{27}$ and $m = 10n$. The edges in a batch are given in COO format, and are unsorted.

Throughput. We evaluate the throughput of our incremental connectivity implementations on all of the input graphs in Table 2, and the two large synthetic graphs generated from the RMAT and BA generators. Table 4 reports the streaming throughput achieved by the fastest variant of each algorithm for each input when all of the edges are treated as a single batch of updates.

As shown in Table 4, the Union-Async algorithm usually achieves the highest throughput. Recall that for static parallel connectivity, Union-Rem-CAS is 1.02x slower on average than Union-Async, but here Union-Rem-CAS is 3.41x slower on average than Union-Async. In the incremental setting, when accessing an element of the *labels* array, the algorithm must check whether the element has been initialized, which incurs an additional overhead: for instance, Union-Async when used in the incremental setting is 1.73x slower on average than when it is used in the static setting. Compared to Union-Async, in Union-Rem-CAS, the *labels* array is accessed much more often, which decreases the throughput.

The other min-based algorithms are much slower than the union-find algorithms because the other min-based algorithms inspect more edges. In particular, Liu-Tarjan is 2.86x slower on average than Union-Async, and Shiloach-Vishkin is 4.87x slower on average than Union-Async.

Throughput vs. Batch Size. Figure 8 shows the throughput of the fastest variant for each algorithm with respect to different batch sizes. When the batch size is small, the performance significantly degrades mainly due to the GPU kernel launch overhead: the time to launch the GPU kernel takes tens of times longer than the time to run it for a small batch size. For small batch sizes, the Liu-Tarjan and Shiloach-Vishkin algorithms are much slower because for each

²We contacted the authors for their code, but were unable to obtain it.

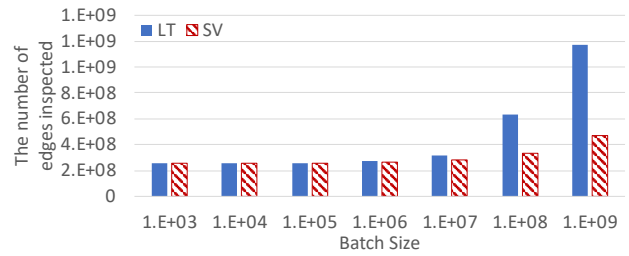


Figure 9: The number of edges inspected for each batch size for Liu-Tarjan and Shiloach-Vishkin on *com-Orkut*.

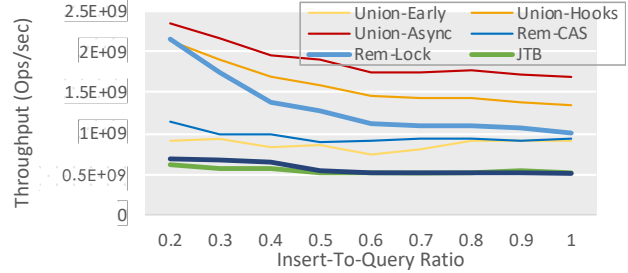


Figure 10: Throughput of the fastest variant (when the be-low ratio is 1) of each algorithm on *europe_osm*, plotted for different ratios of insertions to queries.

batch, the GPU kernels are launched multiple times to process the current batch until the *labels* array converges.

One interesting point is that Liu-Tarjan and Shiloach-Vishkin become slower when the batch size is larger than 10^7 due to the increase in the number of edge inspections. Figure 9 shows the number of edges being inspected for different batch sizes for one variant of Liu-Tarjan as well as Shiloach-Vishkin on *com-Orkut*. The number of edges examined increases significantly for a batch size larger than 10^7 because in those algorithms, all edges in a batch need to be inspected until the batch converges. We observed a similar trend in the other graphs for all variants of Liu-Tarjan as well as Shiloach-Vishkin.

Mixed Inserts and Queries. We evaluate the performance of the variants used in Figure 8 to study how the ratio of insertions to queries affects the throughput. For a ratio of insertions to queries of x , we generate $1/x$ queries with random vertex pairs per insert, and shuffle them with the original edges (which are insertions) that are also randomly permuted to prevent data locality from affecting performance.

Figure 10 shows the throughput with different insert-to-query ratios on the *europe_osm* graph. As the ratio decreases, throughput also increases because the path to the root vertex in the *labels* array is compressed, which speeds up the processing of the remaining insertions and queries. Note that for very well connected graphs (e.g., *com-Orkut*), the throughput with respect to different ratios is quite stable, because even when the ratio is 1, the labels have converged after processing a small subset of insertions.

4.5 Evaluation on Titan Xp (Pascal)

This section presents our evaluation of GConn on the Titan Xp (Pascal) GPU. Table 5 shows the average slowdown against our evaluation on the V100 GPU, as well as the average of the speedup over

Algorithm	coPapers DBLP	cit- Patents	road_usa	soc-Live Journal1	ljournal -2008	delaney _n24	europa _osm	hollywood -2009	kron_g500 -logn21	com- Orkut	indochina -2004	uk-2002	twitter7	com- Friendster	RMAT	BA
Union-Early	8.18e09	6.08e09	5.44e09	5.19e09	3.78e09	4.87e09	6.83e09	4.65e09	4.37e09	5.58e09	2.39e09	3.42e09	3.19e09	1.78e09	3.67e09	7.50e08
Union-Hooks	2.66e10	7.51e09	5.69e09	1.17e10	9.58e09	8.78e09	4.75e09	3.52e10	2.65e10	4.26e10	1.29e10	1.29e10	1.18e10	5.54e09	9.95e09	4.81e09
Union-Async	3.17e10	1.27e10	7.35e09	1.72e10	1.31e10	1.24e10	6.74e09	3.93e10	3.25e10	4.66e10	1.59e10	1.65e10	1.51e10	6.08e09	1.17e10	5.03e09
Union-Rem-CAS	1.05e10	6.50e09	4.46e09	6.20e09	4.00e09	5.70e09	6.05e09	7.64e09	1.03e10	1.16e10	1.98e09	3.27e09	3.80e09	3.14e09	5.60e09	9.61e08
Union-Rem-Lock	3.25e09	5.45e09	3.48e09	1.96e09	4.97e08	5.50e09	4.44e09	6.20e09	1.94e10	4.49e10	4.14e07	1.55e08	5.46e08	7.23e09	1.11e10	4.17e09
Union-JTB	2.78e10	8.70e09	5.29e09	1.33e10	9.39e09	8.63e09	4.68e09	3.32e10	2.52e10	2.94e10	1.14e10	1.21e10	1.41e10	5.85e09	8.14e09	5.70e09
Liu-Tarjan	1.39e10	2.50e09	2.92e09	7.41e09	6.99e09	4.62e09	2.63e09	1.42e10	7.38e09	1.18e10	8.85e09	9.56e09	4.74e09	2.08e09	3.05e09	2.28e09
Shiloach-Vishkin	9.42e09	2.09e09	2.97e09	5.39e09	4.27e09	3.12e09	2.02e09	4.84e09	3.90e09	3.79e09	5.90e09	7.90e09	3.22e09	1.37e09	2.50e09	2.36e09

Table 4: Throughput achieved by incremental connectivity algorithms in GConn on a V100 GPU machine when all of the edges in the graph are treated as a single batch of updates. For each graph, the highest throughput is shown in green.

Algorithm	Average slowdown against V100		Average speedup over state-of-the-art
	Across all variants	Fastest variants	
Static conn.	1.62	1.71	2.38
Span. forest	1.59	1.60	–
Streaming conn.	1.79	2.25	–

Table 5: Summary of results on the Titan Xp machine.

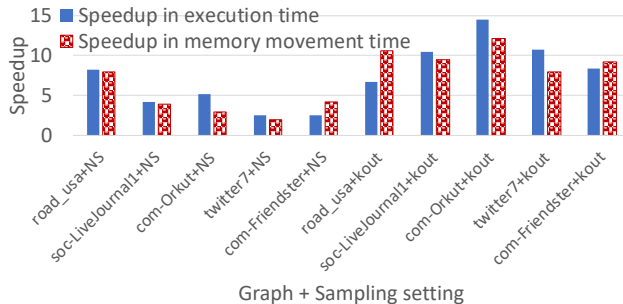


Figure 11: The performance comparison between the fastest variant on the GPU vs. CPU for each graph in the no-sampling setting (+NS) and with k -out sampling (+kout).

the fastest one among GPU-CC, GSWITCH, ECL-CC, and Afforest also run on the Titan Xp GPU. As shown in Table 5, the trends for both GPU machines are very similar, and except for incremental connectivity, the average slowdown is close to the ratio of the bandwidth of the V100 to that of Titan Xp ($900/547.6 = 1.6$). For incremental connectivity, accessing the *labels* array requires spin-locks, which prevents the algorithms from fully saturating the memory bandwidth. We note that we had to modify Union-Rem-Lock to avoid deadlocks [51, 70] for the Pascal machine in which threads in a warp are executed in lock-step [54].

4.6 Performance analysis on CPUs vs. GPUs

In this section, we compare the performance of GConn [29] on the V100 with the performance of ConnectIt. ConnectIt’s experiments were performed on a Dell PowerEdge R930 with 4×2.4 GHz Intel 18-core E7-8864 x4 Xeon processors, a 45MB L3 Cache and a memory bandwidth of 85 GB/sec.

We compare CPU and GPU performance with the five graphs used in ConnectIt that can fit in the GPU memory [29]. To understand the relation between the execution time and the total data movement from/to memory, Figure 11 shows the speedup in execution time achieved by GConn over ConnectIt (blue bars), and the speedup in time for the data movement from/to memory at peak

memory bandwidth using the GPU compared to the CPU (red bars). Our GPU provides $900/85 = 10.59$ times higher memory bandwidth compared to the CPU, but the CPU L3 cache is much larger than the GPU L2 cache. Hence, there can be more memory transactions on the GPU, depending on the graph. As shown in Figure 11, there is a strong correlation between the execution time and the time for data movement (a Pearson correlation coefficient of 0.854). Note that the first five pairs of bars are without sampling, and the rest of them are with k -out sampling. For a fair comparison, we did not apply HB sampling because the best variants in ConnectIt for these graphs always adopt k -out sampling. The best variant of GConn without HB sampling achieves 8.26–14.51x speedup over the best variant with ConnectIt. For incremental algorithms, when we treat all of the edges as one large batch, GConn only achieves 1.85–13.36x speedup over ConnectIt due to the spin-locks on the *labels* array being more expensive on GPUs than CPUs.

To obtain a rough estimate of the monetary cost savings for running the connectivity algorithms on the GPU vs. the CPU, we compare two machines on Amazon EC2: the *p3.2xlarge* configuration, which is very similar to our V100 setup, and the *x1.16xlarge* configuration, which provides a multicore similar to the Dell PowerEdge R930. On-demand pricing of the *p3.2xlarge* and *x1.16xlarge* instances is \$3.06 and \$6.669 per hour, respectively. The fastest variant of GConn is 12.02x faster on average than that of ConnectIt. Hence, for our input graphs, we can expect to save roughly a factor of $(6.669/3.06) \times 12.02 = 26.2$ in costs by using a similar GPU compared to a similar CPU.

4.7 Takeaways and Guidelines

Based on our experimental study, we found that variants of union-find that have not been studied in prior work on GPU connectivity performed the best. We discovered that the sampling phase mostly dominates the execution time, which indicates that reducing the overhead for the sampling phase is crucial for high performance. We found that many of our static connectivity algorithms can be extended to support spanning forest and incremental connectivity, and can achieve high performance as well. Finally, we found GPUs to be a cost-efficient option for connectivity algorithms compared to CPUs, as long as the graph can fit in the GPU memory.

The fastest implementation is dependent on a given input graph, and as GConn supports several hundred variants, trying all variants to find the fastest one can be overwhelming. We provide some guidelines below on how to choose an implementation with high performance.

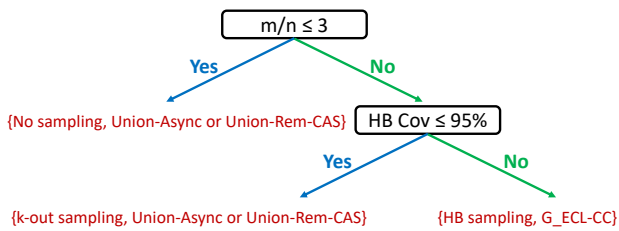


Figure 12: A decision tree for the sophisticated dynamic mechanism.

First, a practitioner can simply use the best variant overall based on our experimental evaluation, which is to use Union-Async or Union-Rem-CAS combined with k -out sampling. For our input graphs, using this approach achieves performance that is within 20.8% on average of the performance of the fastest implementation for each graph.

A more sophisticated dynamic mechanism for selecting sampling methods for a fixed finish method is as follows. First, we observed that if the average degree in the graph is small enough, most edges are inspected in the sampling phase, and an additional overhead to access the edge array multiple times is incurred. Therefore, we found that when the average degree is small, sampling is not a worthwhile optimization. Second, we observed that one does not need to consider BFS sampling, as it is always outperformed by either k -out sampling or hook-based search (HB) sampling. From Table 3, we see that when **HB Cov** (the percentage of vertices in the largest connected component after HB sampling) is high enough, HB sampling is always the best strategy. As shown in Figure 6, in this case, HB sampling samples a minimal number of edges. A rough lower bound of **HB Cov** can be obtained by applying the first step of HB for a few edges, which is inexpensive, and then using this value to choose between HB or k -out sampling.

Our study also provides insights on choosing finish methods that complement a particular sampling method. First, we found that the fastest finish method is always one of {Union-Async, Union-Rem-CAS, or G_ECL-CC}. In the no-sampling setting, either Union-Async or Union-Rem-CAS is the fastest. G_ECL-CC performs fewer on-the-fly compressions, which normally leads to more path traversal (and thus more cache misses). On the other hand, when HB sampling is used, which implies the input graph is very well connected, G_ECL-CC is recommended as only a very few edges are inspected by the union-find algorithms. In this case, G_ECL-CC has the lowest overhead as it does not perform much on-the-fly compression, which requires performing additional writes. On our input graphs, using this strategy for choosing the sampling and finish methods gives a slowdown of only 9.4% on average compared to using the fastest implementation for each graph.

Figure 12 presents a decision tree for the strategy above. **HB Cov** is the lower bound mentioned above, and Union-Async can be substituted with Union-Rem-CAS as their performance is similar.

5 CONCLUSION

We have designed the GConn framework, which supports several hundred efficient implementations on GPUs for static connectivity, spanning forest, and incremental connectivity. To the best of our

knowledge, this paper provides the most comprehensive study of different variants of connectivity algorithms on GPUs. Extensive evaluations show that the best connectivity implementations in GConn significantly outperform other state-of-the-art libraries and implementations. For future work, we are interested in extending our framework to the multi-GPU or distributed memory settings.

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